Learning Abstract Symbolic Mathematics Through Digital Games

by **Afrooz Samaei**

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ABSTRACT

Afrooz Samaei

OCAD University

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Video games are extremely engaging experiences that could offer a productive learning environment to better understand concepts that are challenging to grasp within a traditional classroom setting. Mathematics provides one example of such a subject due to its high level of abstraction. Learning mathematics has long been a challenge for students, and a source of concern for educators. A considerable number of video games have been developed to teach basic mathematical concepts, yet, teaching more advanced mathematics remains a major challenge. This research explored ways to incorporate math education in video games, focusing on modeling concepts from high school mathematics. The goal of this research was to make a series of prototype games that act as extrinsic motivational tools to help students engage more effectively with abstract mathematical concepts. The intent was to help students understand the applications of those concepts by putting them into practice within a game environment.

Keywords: Video Games, Digital Games, Educational Games, Abstract Symbolic Mathematics, High School Mathematics, Learning Through Games, Design Research

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Chapter 1: Introduction

I took a lot of math classes in high school and university. During my undergraduate studies in Geomatics Engineering[1], I faced countless mathematical problems, starting from the very first semester until the end. After a while, I became a master in computing definite and indefinite integrals, solving systems of differential equations, applying the rules of differential and integral calculus to solve related problems, experimenting with numerical methods, etc., and proudly graduated having earned 100% on some of my math courses. Like many other students, I always had this question in my mind: "Would I ever use this in my future career?". But the pressure of heavy school work and the attempt to maintain a high GPA made me ignore this nagging voice in my head, trust the education system, and believe that I would use those concepts sometime in the future.

When I moved to Canada I started tutoring mathematics to secondary and post-secondary students. It started with the intention to generate some revenue, but it soon became a hobby. The joy of teaching, working with young and bright students (who live in a substantially different time and place compared to my generation), and often being challenged by their

[1] Geomatics Engineering is concerned with gathering and processing spatial data and includes fields such as Photogrammetry, Remote Sensing, and GIS.

clever questions made me even more passionate about continuing teaching. I always had a double feeling when teaching: On one hand, I was having such a good time since in the absence of exams and the pressure associated with them I had the opportunity to look at the concepts that I had previously studied from a different perspective, draw new connections, and eventually appreciate what I had learned. But on the other hand, I had a confused and frustrated teenager sitting in front of me, desperately asking for help while mouthing off at her teacher, the Nelson[2], and whoever else that had made her spend her evening on long and tedious mathematical operations.

When I get asked the typical question of “When are we even going to use this?", I think of my own experience and how confused I was when learning mathematics and how I came to appreciate the subject years later in life as I began to use mathematics in different contexts, from simple daily activities to computer programming for games. Of course, I never had to solve differential equations on paper, but I got introduced to a new way of thinking. So, I often reply to my students by saying: You are not going to ever use these concepts directly in your life, the way you are using them now. The real-world problems are much more complex, and you can’t jump into solving them without knowing the basics. You are only learning the foundations of much more advanced subjects and it takes lots of effort, persistence, and patience to master those subjects. In Jordan Ellenberg’s words, it is like weight training to become a professional soccer player. “You won’t see anybody on the field curling a weight or zigzagging between traffic cones. But you do see players using the strength, speed, insight, and flexibility they built up by doing those drills, week after tedious

[2] Nelson is a Canadian Educational Publisher

week" (Ellenberg. 2014, p.2). But how is it possible to convince a teenager to persist in learning such difficult and abstract concepts with the hope of using them at some point in the future?

Having to explain this over and over to my students, I decided to take action instead and do something potentially helpful for this situation. I decided to focus on video games and embed some of the problems in the textbooks within a pleasurable game environment hoping that it would motivate students to continue learning by seeing themselves capable of applying their knowledge to solve problems in a game context. This is the main driving force behind this project.

1.1. Why is it challenging to learn mathematics?

> Knowing mathematics is like wearing a pair of X-ray specs that reveal hidden structures underneath the messy and chaotic surface of the world. Math is a science of not being wrong about things, its techniques and habits hammered out by centuries of hard work and argument. With the tools of mathematics in hand, you can understand the world in a deeper, sounder, and more meaningful way. (Ellenberg, 2014, p.2)

But learning mathematics has long been a challenge and teaching it a major concern for educators. Some countries such as the United States with high rates of technological and industrial advancement are constantly voicing concern regarding the poor performance of their students in science and mathematics contests on national and international levels (National Research Council[3], 2011). This is worrisome since a lack of high-achieving science students today could constrain the future scientific and technological workforce and threaten a country's innovations, economic growth, and international competitiveness

[3] Referring to the National Research Council of the United States throughout this paper.

which depends on proficient scientists, engineers and technical workers (National Research Council, 2011). Future generations need to be prepared to innovate and constantly adapt to and create new technologies. If they are going to have any hope of a secure future, they need to be tech-savvy: "…thinking that learning math and science and mastering new technology are completely natural, normal, and nonthreatening …because today science and technology are part of everything we do" (Gee, 2006).

When students are asked about their lack of interest in mathematics they often reply with a similar set of statements: It is hard; It is not relevant; It is useless. As a matter of fact, students are right when they complain about having to do long and seemingly meaningless algebraic operations over and over. First of all, mathematics is indeed hard to learn.

> It is a way of thinking that the human brain finds unnatural. Our brain is a marvel in the animal kingdom, but its great power lies in language, pattern recognition, reasoning by analogy, and the ability to make rapid decisions based on little information. It is particularly ill suited to the methodological, linear reasoning and total precision of mathematics. We have to train our minds to do math, and it's hard work. (Devlin, 2011, p.176)

Apart from this, schools do not seem to be doing a great job in helping students with this issue. Classrooms have not adapted to the significant growth of the information technology over the past years in order to train students through more meaningful activities, rather than memorizing discrete facts and acquiring isolated skills (Shaffer et al., 2005). In addition, performance on timed tests is highly overrated to a degree that students often miss the actual purpose of studying science and mathematics, which is applying them to real situations in life. In Salman Khan's words, these tests offer nothing more than a "snapshot" of where the student stands at a given moment in time, without giving a chance for a second or third attempt. They fail to say anything about how long learning will be retained and how deeply a concept is understood (Khan, 2012, p.92). Most of the students who pass these tests cannot apply their knowledge to the real world and use the rote learning and standardized skills they got in school to think in innovative ways (Gee, 2006).

With this approach in education, students no longer see science as connected to the real world and lose interest in the subject. Our technologically impoverished schools can no longer meet the need of today's kids who are immersed in digital media and accustomed to learning through digital technologies (Gee, 2006). It is important for researchers, educators, and digital content producers to continue their effort to enhance the learning experience and enrich its quality to prepare the future generations.

1.2. Learning Through Simulations and Games

Over the past thirty years with the creation of simulated environments and digital games, the potential of these technologies in education and training have started to be realized. Simulations are computational models and representations of real situations or phenomena that allow users to interact with and manipulate objects and experiment with modifying different parameters (Clark et al., 2009). They are great tools to learn and practice activities which are challenging, dangerous, or expensive to learn and practice in a real-world environment. Airlines train their pilots, NASA trains its astronauts, medical schools train surgeons, and armies train soldiers by putting them in simulations (Devlin, 2011, p. 3).

Video games typically involve simulated environments and, just as simulations do, allow interaction and exploration, but are different from simulations in several ways. Apart from provoking certain levels of play, engagement, and enjoyment, games generally incorporate rules and explicit goals, along with an accompanying reward system to track players' progress (Clark et al., 2009). It is true that simulations could also provide engagement and enjoyment for some users, with certain tastes and interests, in addition to enticing them to define their own rules and goals. For instance, *The Sims* video game could be regarded as a simulation, rather than a game, but players often create goals and challenges, similar to those in a game (Clark et al., 2009). Although drawing a clear border between the two technologies is hard, Clark (2009) lists the following features as main characteristics of video games: (i) Games allow users to make choices that affect the state of the game; (ii) Games involve an overarching set of explicit goals with a system to measure progress; (iii) games provide subjective opportunities for play and engagement.

A great deal of research has been dedicated to studying the effects of video games on learning that mainly praise video games as interactive experiences that can provide a productive learning environment to better understand concepts that are challenging to grasp within a traditional classroom setting (Coller, 2016; Devlin, 2011; Gee, 2007; Shaffer, 2006). Many researchers have sought to prove that a functioning game or simulation can engage students in inquiry and enhance motivation (Clark et al., 2009). Yet according to the National Research Council (2011), "…only a few studies clearly articulate the learning goal of the game; the theory of action about how the goal will be advanced; and the measures, analyses, and data used to assess learners' progress toward the goal. Most studies lack control groups, making it difficult to conclude that the game or simulation caused any learning gains observed among the study participants."

The focus of this research is to build a series of digital game prototypes that help the player learn and practice abstract symbolic mathematics. A considerable number of video games have been developed to teach students basic mathematical concepts, and some have proven to be influential in terms of improving students' number sense and mathematical thinking abilities (Kiili et al., 2015). However, teaching more advanced mathematics remains a major challenge. The reasons behind this issue will be discussed in more depth in section 2.4. This thesis intends to explore ways to incorporate high school mathematics into video games. Considering the scope of this master's thesis, the focus would be narrowed down to one subject: Modeling with Quadratic Functions. The goal is to help students practice writing mathematical models to solve related problems that arise from real-life situations, presented within a game environment.

The main question that this research hopes to answer to is:

- How could digital games be used to help secondary school students practice applying abstract symbolic mathematical concepts described in their textbooks to solve problems?

To answer this question this research will present an overview of the literature with the goal of understanding the principles of learning embedded in good video games, common frameworks for designing games, recent research on learning mathematics through video games, and some examples of educational games. The main methodologies incorporated in this research are *design-oriented research*, *iterative design,* and *playtesting*, which will be discussed in chapter 3. To support this document several prototypes have been developed and user tested. These prototypes and the results of user testing will be presented in detail in chapter 4 and 5.

It is important to mention that this research is not suggesting new learning theories in mathematics. Creating new ways of teaching the subject is beyond the expertise of the author and the scope of this thesis. The intention is rather to provide a new and potentially better environment to practice those concepts and understand their applications while following the teaching guidelines provided by the Ontario Ministry of Education. In addition, considering the relatively short amount of time that this research is being conducted in, it would not be realistic to rigorously assess the final results regarding the learning outcomes. Understanding whether the final product has positive effects on students learning requires running more studies, with a larger group of participants, using control groups, over a longer period of time. The final product will be assessed based on the players' feedback regarding their experience of playing the game and solving the mathematical problems.

Chapter 2: Contextual Review

This research focuses mainly on literature from scholars who have explored educational aspects of video games. James Paul Gee, an American educator and researcher, is a leading advocate for learning through video games and a well-known scholar in this field. In his book, *What video games have to teach us about learning and literacy*, he discusses principles of learning built into good video games. He describes each principle in a way that is relevant to learning both in classrooms and a well-designed video game. Some of these principles will be described in the following sections. These principles are selected based on their relevance to this research project and the methods used to develop the prototypes. Keith Devlin, a mathematician who is engaged with developing math educational video games, has focused specifically on using video games in math education. He is cited quite often in this section due to his notable works which include both theory and practice. In addition, the report *Learning Science Through Computer Games and Simulation,* published by the National Research Council (NRC) of the United States, in 2011, has been investigated as it reviews the available research on learning science through interaction with digital simulations and games, and provides guidelines for academic researchers and developers who are engaged with this field. Moreover, some developed products will be explored. The works chosen are developed by Brianno Coller, a university professor who uses video games to teach subjects to mechanical engineering students, and Triseum, a Texas-based company producing educational video games.

2.1. Why Video Games? – Principles of Learning in Good Video Games

2.1.1. Situated Meaning and Experiential Learning

Experiential learning theory, which emphasizes the role of experience in human learning and development, was formed mainly based on works from twentieth century scholars, such as John Dewey, Kurt Lewin, Jean Piaget, and others, to develop a holistic model of the experiential learning process and a multilinear model of adult development (Kolb & Kolb, 2005). Experiential learning theory defines learning as "…the process whereby knowledge is created through the transformation of experience. Knowledge results from combination of grasping and transforming experience" (Kolb 1984, qtd. in Kolb & Kolb, 2005). This theory indicates that learning is best conceived as a process, "…a continuing reconstruction of experience" (Dewey, 1897, qtd. in Kolb & Kolb, 2005), not in terms of outcome. The focus should be on the process of learning and students should be given feedback on the effectiveness of their learning effort. This learning process is in fact "relearning", which means that students' previous knowledge, beliefs, and ideas about a topic are examined, tested, and integrated with new, more refined ideas. Learning is a holistic process of adaption to the world and, in addition to cognition, it involves the integrated functioning of the total person – thinking, feeling, perceiving, and behaving (Kolb & Kolb, 2005).

In contrast to experiential learning theory, most current educational systems practice a "transmission" model, where pre-existing fixed ideas are transmitted to the learner (Kolb & Kolb, 2005). According to Gee, this view of learning "…stresses the mind and not the body"

(Gee, 2007, p.71). It treats the human mind like a digital computer; a device that operates by rules and manipulates symbols that have no meanings to it. This view defines learning as a matter of generalizations, principles, rules, abstractions, and logical computations (Gee, 2007, p.71).

Gee promotes a view of learning that stresses the experience that humans have gathered in their lives. People store experiences, edit them based on their interests, values, goals, and socio-cultural memberships. When people are faced with a new situation or are solving a problem, they start reflecting on their previous experiences. Sometimes their previous experience can be applied to the new situation as is, and other times they should be modified to adapt to the new situation. The learning happens in this process of adapting past experiences to new situations in the future (Gee, 2007, p.71-72). As Gee points out, "…human thinking is deeply rooted in embodied experience of the world. …Human learning and thinking builds abstraction on the basis of concrete images from embodied experience of a material world" (Gee, 2007, p.72). Unfortunately, what happens regularly at schools is that many concepts are taught through words, symbols, and abstractions without clarifying their connections to images or situations in students' embodied experiences of the world (Gee, 2007, p.72). Schools should avoid focusing solely on presenting general, purely verbal meanings to students; meanings that a person has no ability to customize for specific situations and that offer the person no invitations for embodied actions in different situations (Gee, 2007, p.83).

Gee claims that video games, as opposed to most schools, encourage situated, experiential, and embodied forms of learning and thinking (Gee, 2007, p.73). The way that subjects such as math and sciences are taught at school is that words and meanings usually float free of material conditions and embodied actions and they take only general meanings (Gee, 2007, p. 84). Students would not be able to make sense out of what they learn at school unless they experience the meaning in a situated and embodied way. If learners see algebra, for instance, spelled out in more specific material situations, such as Galileo's principle of motion, they can master it in an active and critical way, not just as a set of symbols to be manipulated in a passive and rote manner on tests. (Gee, 2007, p. 87).

In the context of learning abstract mathematics, the ideas of situated meaning and experiential learning becomes much more crucial. Abstract systems originally got their meanings through embodied experiences for those who really understand them. Abstraction rises gradually out of the ground of situated meaning and practice and returns there from time to time, otherwise it is meaningless to most people (Gee, 2007, p. 87). So, it is important to show students how an abstract concept arises from a concrete situation. This has been the main objective in developing prototypes to support this research. As Devlin suggests, in an ideal learning environment the mathematics to be learned must arise naturally in the environment and have meaning in it (Devlin, 2011, p. 25). Hence, the intention when developing prototypes has been to create a set of problems that are embedded in the game world and have meaning within the game environment. These problems illustrate how certain abstract concepts gain meaning within a concrete situation and provide a scenario where students are required to dig into their knowledge of algebra and to put it into practice in order to be able to solve the game's challenges.

2.1.2. Forming Hypotheses

Playing video games allows the player to take some actions, reflect on those actions based on the consequences in the game world, and refine future actions. Gee describes that this reflective practice is the basis of how humans learn, considering the human mind as a powerful pattern recognizer. For instance, when a young child tries to crush a soft cloth book, she forms a hypothesis about the book by unconsciously reflecting on her actions and recognizing a pattern, say: "Books are soft, they squish, but don't break". When the child tries to squish a paper book, based on her previous hypothesis, she reforms a new hypothesis by reflecting on this new experience of interacting with a book, say: "cloth books squish and paper ones tear" (Gee, 2007, p. 89).

So, a good learning environment should engage the player in such a process, as described in the following four steps (Gee, 2007, p.88):

1. The player must *probe* the virtual world (which involves looking around the current environment, clicking on something, or engaging in a certain action).
2. Based on reflection during and after probing, the player must form a *hypothesis* about what something (a text, object, artifact, event, or action) might mean in a usefully situated way.
3. The player *reprobes* the world with that hypothesis in mind, seeing what effect he or she gets.
4. The player treats this effect as feedback from the world and accepts or *rethinks* his or her original hypothesis.

Schools are responsible for providing students with embodied experiences in and through which they can form networks of associations that must continually be rechecked against the world. The role of teachers is very important in this process. Students need active teachers who can guide the hypotheses they form and the patterns they recognize. Otherwise, students might rely excessively on their own creativity and form patterns that in the end are not reliable (Gee, 2007, p. 92).

In the final prototype developed for this project, the intention has been to design the environment and challenges in a way that players could form hypotheses when faced with a challenge, and later refine and build upon their previous hypotheses to solve a new, more complicated challenge. But unlike most games, this is not done through a direct action-consequence loop, but via a support interface that helps the learner extrapolate mathematical concepts to solve the next challenges. The interface provides a connection between the game world and the textbooks as it contains instructions on the process of modeling with quadratic functions. Instructions have been provided to guide the players along a path to solve the problems, which prevent them from forming hypotheses that might lead to a dead-end.

2.1.3. Experimenting and Risk Taking

A good learning environment should allow the learners to experiment with different techniques and explore new ways to solve a problem. Through this exploration process, learners often make mistakes. In Devlin's words, from a learning perspective they are not actually mistakes; rather "…they are choices that subsequent experience tells us were not correct or optimal" (Devlin, 2011, p. 79). However, failure should be properly designed so it can be an effective learning mechanism. It must be big enough and hurt sufficiently for us to avoid repeating it but should not be so great that we lose interest in experimenting (Devlin, 2011, p. 79). Video games provide an opportunity for players to learn by exploration. Players can take risks, but within a safe environment and in absence of the real-world costs and consequences.

In the case of mathematics, learning from mistakes becomes very important, as even professional mathematicians make mistakes most of the time if they are working on hard enough problems (Devlin, 2011, p. 87). As Devlin describes:

> A feature of mathematics that makes it psychologically difficult to learn is that in many cases, if a student fails to get the correct answer to a question, they are simply wrong. No half measures here; wrong is wrong. Given that degree of finality, it doesn't take many cases of being wrong to persuade some people to give up altogether; thinking they are simply not cut out for the subject. (Devlin, 2011, p. 87)

To incorporate this principle into the prototypes the challenges are designed in a way that invite the player to experiment with and try multiple solutions to overcome the challenges. In addition, there is no single solution to a problem and players can pass an obstacle by trying multiple numbers. However, the players are eventually rated based on how optimal their solutions are. In other words, there is no state of failure in the game, as opposed to traditional exams and questions in the textbooks for which there is only one right answer. Players can pass a challenge without failing, but they are encouraged to perform some mathematical analysis to find an answer that is more optimal.

2.1.4. Material Intelligence, Manipulation, Distributed Knowledge

Video games are made of environments, objects, tools, and technologies that can store knowledge. Many virtual objects in a game environment are 'smart tools', which have certain information that the player does not have. Hence, combining these information with those of the player extends her abilities (Gee, 2005). Similarly, students in a classroom become more powerful actors by learning to integrate their own knowledge with the knowledge built into their smart tools. In the case of educational games, the real-world player and the virtual objects combine their skills and knowledge that is constitutive of a certain type of professional practice: “This frees learners to engage their minds with other things while combining the results of their own thinking with the knowledge stored in these tools, technologies, material objects, and the environment to achieve yet more powerful effects” (Gee, 2007, p. 110)

In addition, games offer the player the ability to easily and effectively manipulate the world’s objects, objects which become tools for carrying out the player’s goals: “Humans feel expanded and empowered when they can manipulate powerful tools in intricate ways that extend their area of effectiveness” (Gee, 2005).

In the main prototype developed during this research, a graphing interface has been incorporated in the game. When students follow the instructions to solve a challenge and complete the required fields, the related mathematical functions are automatically graphed within the interface based on the players’ inputs. This removes the burden of manually graphing the functions from the player. In addition, it provides a visual representation of the situation that could potentially help the player better solve the problem. Players can manipulate the graph and experiment with different shapes by changing the mathematical equations. This interface bridges the game environment to the textbooks by providing information and instructions that the student might need to solve the problems.

2.1.5. Individualized and Self-Directed Learning

According to Khan, one of the main issues with the current education system is that disciplines are divided into subjects, and the subjects are further divided into independent units, as if unconnected. A consequence of such approach is that teachers might not get a chance to cover a subject thoroughly since our schools measure out their efforts in increments of time rather than in target levels of mastery. When the interval allotted for a given topic has run out, it is time to give a test and move on to the next subject (Khan, 2012, p. 83). This way, students become a victim of what Khan calls "Swiss cheese" learning; "Though it seems solid from the outside, their education is full of holes" (Khan, 2012, p. 85). Students can pass a course by getting a mark of 75 percent and continue to take the next course which builds on previous materials. A mark of 75 percent, however, indicates that the students miss one-quarter of what they need to know. This is like building a house on 75 percent of a foundation (Khan, 2012, p. 84). The structure can go up, but it will eventually collapse. We rush through conceptual modules and pronounce them finished when in fact only a very shallow level of functional understanding is achieved and before the applications of the concepts in the real world is realized (Khan, 2012, p. 89). This clarifies the importance of tools that enable students to practice based on their own pace, outside of the classroom time.

Video games enable students to repeat a task until mastery is achieved and before moving on to the next level. In addition, there are multiple ways to progress in a game, which invite the players to use their own skills and rely on their own strength to solve a problem, in addition to experimenting with alternative approaches (Gee, 2007, p. 110). Moreover, video games provide continuous feedback as the player moves on and signal the learner's ongoing achievement (Gee, 2007, p. 64).

Lastly, it is very important that learners understand why they need to learn a subject. As Malcolm Knowles indicates, "If we know why we are learning and if the reason fits our needs as we perceive them, we will learn quickly and deeply" (qtd. in Khan, 2012, p. 175). In a situation where we do not *have to* learn a subject, rather we *choose to* learn it, the motivation behind learning serves to focus our attention and thereby make learning easier (Khan, 2012, p. 176). Video games create challenges for players that require them to have certain skills. These skills might be simple to achieve within the game, for instance by repeating a task. But sometimes the players need to improve their knowledge base using the information available outside of the game world. Hence video games could potentially give players motivation to learn and provide reasons for gaining certain skills.

2.2. Game Design Frameworks

Game designers, researchers, and scholars often use various frameworks to develop, analyze or criticize a game. The MDA framework, which stands for Mechanics, Dynamics, and Aesthetics, is one of the formal approaches to understanding games and attempts to eliminate the gap between game design and development, game criticism, and technical game research (Hunicke et al., 2004). Using the MDA framework games can be viewed and analyzed through three separate but causally linked lenses: Mechanics, which are the multiple actions, behaviors, and control mechanisms the players have access to; Dynamics, which are the mechanisms that create certain feelings during the gameplay; Aesthetics, which describe the desirable emotional responses created for the player while playing

(Hunicke et al., 2004). Another framework to study games is proposed by Salen and Zimmerman which analyzes games through three "schemas": Rules, which focus on algorithms and mathematical structure of games; Play, which focuses on players' interaction with the game and other players; Culture, which emphasizes the cultural context the game is embedded in (Salen & Zimmerman, 2004, p. 102).

In addition to these, many researchers have tried to develop frameworks specific to designing educational games. It is extremely challenging to design and develop an educational game with reliable learning content while maintaining the entertainment aspects. So, good design methodologies and frameworks are needed to help with this goal (Ibrahim & Jaafar, 2009). Annetta defines "six I's" as an approach for designing *serious educational games* – games that are not designed for commercial purposes, and they rather target K-20 content knowledge and allow teachers and learners to connect real-world scenarios with textbook content and provide reasons for learning a subject (Annetta, 2010). The six components that Annetta describes are:

> **Identity:** Taking an identity through an avatar, for instance the identity of a scientist, entices the players to be more invested in an activity.
>
> **Immersion:** Being immersed in the game's environment leverages players' sense of presence, engagement in the content, and motivation to overcome the game's challenges.
>
> **Interactivity:** Player's communication and team work with other players or computerized agents are important to the learning process.
>
> **Increasing Complexity:** As the game progresses, the player's abilities should improve and hence, the game's challenges should become more difficult. If the machine understands the strengths and weaknesses of the user, then the environment can adapt accordingly to arrive at the goal of learning.

Informed Teaching: The recorded behaviors and gathered data during the game provide an opportunity for *virtual observation*, which is like physically observing students' performance in the classroom. This allows the teachers, or the game's artificial intelligence, to adjust game's scaffolds and develop other activities.

Instructional: For games to be instructional they should provide opportunities for players to be challenged, help them adapt to the challenges, and predict ways to circumvent other challenges. Games need to be organized, so that players can effectively recall their previous information and connect them to the new experience they take in the game, allowing players to assimilate the embedded content.

The development of the prototypes in this research project was mainly led by the mathematical challenges embedded in the game. The main purpose of developing the prototypes has been to help students understand some of the applications of mathematical concepts (in this case, modeling with quadratic functions) that they learn from the textbooks, by embedding problems within a game world that has certain connections to the physical world. The goal is to help students understand an abstract concept by connecting it to a concrete situation. Hence, the mathematical challenges and the game world play an important role in the game design process. For students to draw connections between the events in the virtual game world and their understanding and experiences of the real physical world, I found it helpful to build a game environment that is like the physical environment, yet, is simplified enough to match a grade 12 student understanding of mathematics.

Since my focus has been centered on developing the game world, the main effort has been to design the environment in a way that challenges arise from it, with increasing levels of difficulty, and to design the mathematical instructions. Focusing deeply on the game rules, the concept of identity, or cultural contexts were not priorities of this research. There are various elements involved in designing an educational video game and multiple approaches to design, as discussed above. Yet, covering all those elements requires research over a longer period than available. The concept of *identity*, for instance, is an important topic in designing video games which has been explored by many scholars (Annetta, 2010; Devlin, 2011; Gee, 2007; Shaffer, 2006). However, focusing on identity and exploring research and theory in psychology, in addition to education, is outside of the scope of this thesis. The primary focus of this research has been embedding the mathematical challenges in the game world and designing the levels of difficulty. Hence, there has been no avatars used in the prototypes.

Since the focus has been mainly on the game world, the visual assets were chosen very carefully to ensure that the environment is visually appealing, and that the player enjoys navigating it. According to Jenkins (2004), "…game consoles should be regarded as machines for generating compelling spaces, that their virtual play spaces have helped to compensate for the declining place of the traditional backyard …and that the core narratives behind many games center around the struggle to explore, map, and master contested spaces". By creating a game world that we can navigate and interact with, the spaces in video games have the potential to evoke narratives, remediate an existing story, and give concrete shape to our memories. Spatial design can enhance our sense of immersion within a world (Jenkins, 2004). It was hoped that by building a compelling and familiar game environment, players become motivated to navigate the world and encounter the mathematical challenges and can draw connections between the application of certain concepts within the game and the physical world.

2.3. Recent Research on Math Educational Games

There are a considerable number of math education video games developed (DimensionM, DragonBox, Timez Attack, etc.), but as Devlin argues most of these games have been largely focused on trying to develop mastery of basic skills (doing arithmetic operations for instance) rather than mathematical thinking (Devlin, 2011, p. 2). Devlin distinguishes mathematical skills from mathematical thinking and emphasizes the importance of mathematical thinking and defines it as "…a whole way of looking at things, of stripping them down to their numerical, structural, or logical essentials, and of analyzing the underlying patterns" (Devlin, 2012). When mathematicians are faced with a complex problem, they often simplify the situation by eliminating the unnecessary details and focusing on the core elements of the problem, such that their solution to the simplified version of the problem is precise enough to be used to solve the original complex problem. Despite this important skill, what students are often taught at school is to follow certain mathematical procedures, meaningless to them most of the times, rather than learning how to think mathematically.

> Many people, even those in positions of power and influence, don't understand what mathematics is and how it works. All they see are the skills, and they think – wrongly – that mathematics is about those skills. Given that most people's last close encounter with mathematics was a skills-based school math class, it is not hard to see how this misconception arises. (Devlin, 2011, p. 2)

The type of mathematics taught in school influences conceptions about mathematics. Many people, including video game developers, have experienced a type of math instruction which is based on isolated facts, procedures and memorization. Hence many digital math apps emphasize these skills, along with speed, without comprehensively attending to other aspects of mathematical proficiency (Pope & Mangram, 2015). For instance, many video games focus simply on addition, subtraction, or multiplication algorithms without intending to improve number sense – that is "…being mathematically proficient with numbers and computations. It moves beyond the basics to developing a deep understanding about properties of numbers, and thinking flexibly about operations with numbers" (Pope & Mangram, 2015).

Moreover, many of the digital games developed so far fail to define clear learning goals. According to the NRC (2011), an important design feature of educational games should be defining and targeting one or more specific learning goals, before considering other features. Research shows that not focusing on one clear goal and minimizing the irrelevant cognitive demand could distract students from the primary learning goal. Defining these goals is equally important when it comes to assessment and measurement of the effectiveness of a game (National Research Council, 2011).

The NRC identifies *motivation* and *conceptual understanding* as two valued goals of informal science learning and uses these as a framework to identify the learning goals of a video game. According to the NRC, the body of research about the effectiveness of games in supporting science learning is very limited and inconclusive. Most studies lack a control group for making comparisons between learning through games and other forms of instruction. The research in this field is beginning to emerge and the evidence shows that games can motivate the player to learn and improve conceptual understanding (National Research Council, 2011).

According to a report published by the NRC in 2001, called *Adding It Up: Helping Children Learn Mathematics,* conceptual understanding is identified as one of the strands of mathematical proficiency and is defined as "...comprehension of mathematical concepts, operations, and relations … [It] refers to an integrated and functional grasp of mathematical ideas. Students with conceptual understanding know more than isolated facts and methods. They understand why a mathematical idea is important and the kinds of contexts in which it is useful" (Adding It Up, 2001, p. 118).

Devlin refers to the conceptual understanding of mathematics as a skill that is indeed hard to achieve and one that happens over a long period of time. In addition, it is very difficult to understand whether students have sufficiently acquired this skill. He argues that although empowering students with a conceptual understanding of subjects should be the educators' goal, it is important to understand that it cannot be guaranteed, and it does not happen over a short period (Devlin, 2011, p.115). He refers to the use of the term 'integrated and functional grasp' by the NRC when it defines conceptual understanding and argues that a goal of math learners should be achieving what he calls *functional understanding*, which means understanding a concept sufficiently well to get by for the present. He believes that the NRC's definition of conceptual understanding suggests "…an acceptance that a realistic goal is that the learner has sufficient understanding to work intelligently and productively with the concept and to continue to make progress, while allowing for future refinement or even correction of the learner's understanding in the light of further experience" (Devlin, 2011, p. 115).

2.4. Challenges of Learning Advanced Mathematics Through Games

As students start to encounter abstract symbolic mathematics, such as algebraic concepts, starting at Grade 9 in Canada, it becomes much harder for them to grasp those concepts and, more importantly, learn how to connect them to real-life events and their previous experiences. People tend to prefer the concrete to the abstract. Psychologists and anthropologists believe that we are not born with an ability to understand abstraction, but we can acquire it, with lots of effort, during the process of our intellectual development (Devlin, 2011, p.109). Another reason why many people struggle with learning mathematics is the inadequate means that have been historically used to teach mathematics (Devlin, 2011, p. 48).

> For over 2000 years, the only way to provide mathematics education to everyone was through the written word – textbooks. …Mathematics is not about acquiring basic skills or learning formulas. It's a way of thinking about problems in the world. …Math is not a body of knowledge, it's something you *do*. And the printed word can be a terribly inefficient way to learn how to do something. (Devlin, 2011, p. 2)

Devlin emphasizes the importance of *learning by doing* when it comes to learning mathematics, as opposed to solely relying on textbooks and practicing on paper. The learning environment should provide instances of how a mathematical concept would be used, along with sufficient variation in circumstances of use, rather than introducing decontextualized and abstract concepts, which is why many people struggle with symbolic mathematics (Devlin, 2011, p. 27). The main goal of mathematics education is to enable the learners to use the concepts in many different situations and make associations between those concepts and real-life events. Devlin argues that digital games could be an ideal medium to provide an environment for such goal as they allow experiential learning, and as such the players can safely experiment with different ways of solving a problem.

Devlin claims that although video games are ideal tools for learning *everyday mathematics*, the kind of mathematics performed mentally in a real-world context, it is much more challenging to provide learning for abstract symbolic mathematics, the kind usually performed using pencil and paper, through video games.

> …because their underlying educational philosophy is situated learning, they do not lend themselves naturally to teaching abstract symbolic math such as algebra. …Many of the concepts of advanced mathematics (roughly calculus and beyond, though the transition occurs prior to that) are linguistically constructed and have no natural real-world meanings. This is not to say that the concepts cannot be applied to the real world. Indeed, in many cases that is precisely why they were developed in the first place. But those and other advanced math concepts are created through the symbols used to represent them. In my view, you cannot …effectively construct them from more basic concepts. Their meanings should be bootstrapped within mathematics, and that means there is no alternative to mastering them than to first learn the formal definitions and the symbolic manipulation rules, then use them repeatedly, at first without understanding them, in different mathematical contexts, until their meaning emerges. (Devlin, 2011, p. 153-155)

For instance, students of physics, engineering, or economics need to learn certain concepts by the end of their freshman year or they will not be able to proceed. Within the limited time they have, they would not be able to acquire a conceptual understanding of what they learn, but it is possible to achieve some degree of *functional understanding*; "…namely, knowing when and how to apply each technique and what its limitations are" (Devlin, 2011, p. 155). Devlin indicates that although it is challenging to provide an opportunity for students to learn abstract symbolic mathematics directly through a video game, it is possible to provide a situation where the learning takes place outside of the game, and use the game to provide initial reasons for learning and then using the math, which is carried out offline (Devlin, 2011, p. 171). It is very difficult to embed advanced mathematics into a game without compromising on the quality of the gameplay. The concepts of abstract symbolic mathematics, such as algebra, and solving related problems require a long process of thinking and reflecting. "The essence of algebra is to step back from the everyday world, reflect on it – often at length – and think abstractly across many real-world situations, perhaps coming up with a general formula that can be applied repeatedly in different real-world situations" (Devlin, 2011, p. 167). This is not a desirable process when playing a video game, as it separates the player from the game world and might negatively affect the gameplay experience. However, the game itself could provide incentives for the player to step outside of the game and acquire certain skills, to be able to make progress in the game. Hence, it is possible to entice the player to "do the math" outside of the game, to perform the mathematical operations that have a direct impact on the gameplay, to succeed in the game (Devlin, 2011, p. 168). To cite an example, Devlin refers to a video game developed by a university professor, Brianno Coller, for a numerical methods course for mechanical engineering students, which will be described in the next section.

Based on the discussions in previous sections, the learning goals of the prototypes in this thesis has been to increase motivation to learn, in addition to help players acquire a functional understanding of mathematical concepts, in this case writing models with quadratic functions. To understand whether the prototypes meet these goals user testing is required. The latter goal is more difficult to assess through playtesting only and requires more rigorous tests using control groups, which is outside of the scope of this thesis.

Bearing in mind the challenge tied to incorporating advanced symbolic mathematics into video games, as discussed above, the intention of this research is to present an example of such concepts situated in a game environment and show the players how it can be used to solve a related problem.

2.5. Examples of Educational Games

2.5.1. NIU-Torcs

Engineering courses typically offer a high level of intellectual intensity, in which students feel that materials are challenging and important (Coller & Shernoff, 2009). One of the core courses in the undergraduate mechanical engineering curriculum, in Northern Illinois University, has been taught through a computer game, since 2005. In this numerical methods course, all assignments and learning experiences are built around a computer game. Brianno Coller, who is a professor in the mechanical engineering department in Northern Illinois University, took the video game *Torcs* and modified it for his class to build *NIU-Torcs*. NIU-Torcs is a car simulation game in which the player (the student) writes the algorithms for all the car movements, before being able to play. The car is motionless at the beginning of the game. Students are given the task of writing a computer program to give the car its driving commands, such as pedals, gear, and steering wheel adjustments, and then race it around a track. In doing so, students learn and implement numerical methods content, become able to see the results of their algorithms, and learn the related concepts by iteratively refine and test their solutions. The problem embedded within the game is different in many ways from those in the textbooks. It arises authentically through an engineering problem and allows the students to think, act, and value like an engineer (Coller & Shernoff, 2009).

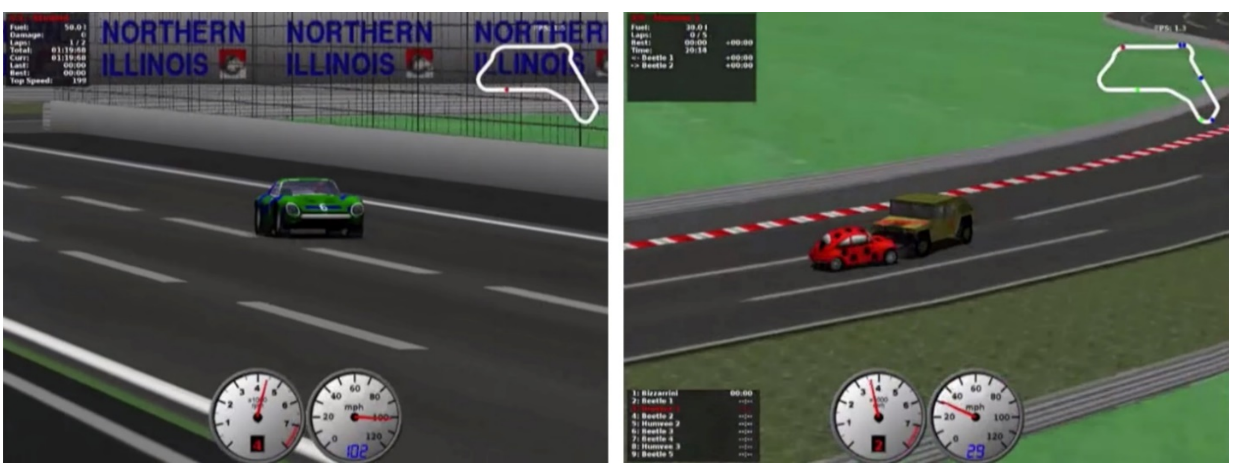

Figure 1- NIU-Torcs

Source: “Rethinking Engineering Education with Video Games | Brianno Coller | TEDxNorthernIllinoisUniversity.” *YouTube*, uploaded by TEDx Talks. 21 June 2016, www.youtube.com/watch?v=tEacOUaViHA&t=257s.

Coller and Shernoff indicate that the video game approach significantly increases the level of challenge and concentration, compared to the traditional homework and class work. Students face a challenge that takes nearly fifteen weeks to solve, and they normally experience a high level of intellectual intensity. But on the other hand, adding the experiential characteristics of active leisure pursuits and enabling the students to tinker with the car make them feel active, engaged, and interested, possibly because goals were clear and feedback about the performance was immediate and free-flowing. Students experience higher levels of intrinsic motivation, enjoyment, and interest, and generally feel a sense of accomplishment, mainly because of writing a computer program that has meaning. Feeling more creative and less worried are also hallmarks of peak engagement during active leisure pursuits (Coller & Shernoff, 2009). By being engaged with NIU-Torcs, students experienced more *concentration*, *interest*, and *enjoyment*, which are the emotional ingredients that foster optimal learning (Shernoff et al., 2003, qtd. in Coller & Shernoff, 2009).

> In more than ten years of teaching engineering, the lead author has never seen so many students eager to learn and eager to take on difficult challenges as he has in the game-based numerical methods course. He has never seen so many students bring their parents, siblings, and friends outside of engineering into the lab to show what they have been doing. The lead author has been surprised to see so many students create videos of their cars in action to show to prospective employers. (Coller & Shernoff, 2009)

2.5.2. Variant: Limits

Variant, developed by Triseum, is a series of educational video games designed to "…assist students with the intuitive understanding of calculus concepts" ("Variant: Educational Game - Bringing Calculus to Life"). The first game of this series was published in 2017 and is called *Variant: Limits. Variant: Limits* is a third person game, in which the player explores an immersive three-dimensional environment and solves puzzles, such as unlocking a door to enter a new zone, by answering to questions related to Limits subject in calculus. When the player faces a challenge, an interactive interface (figure 3) appears on the screen which allows the player to find the right answer and then test it to see if the desired goal is achieved in the game world, for instance if the door is unlocked. Solving each challenge unlocks further parts of the game world which includes new concepts and lessons.

Figure 2- Variant: Limits

Source: Dickeson, Linda. "Engaged Calculus with Award Winning Variant: Limits." *USDLA*, 12 July 2017, www.usdla.org/blog/2017/07/12/engaged-calculus-with-award-winning-variant-limits/.

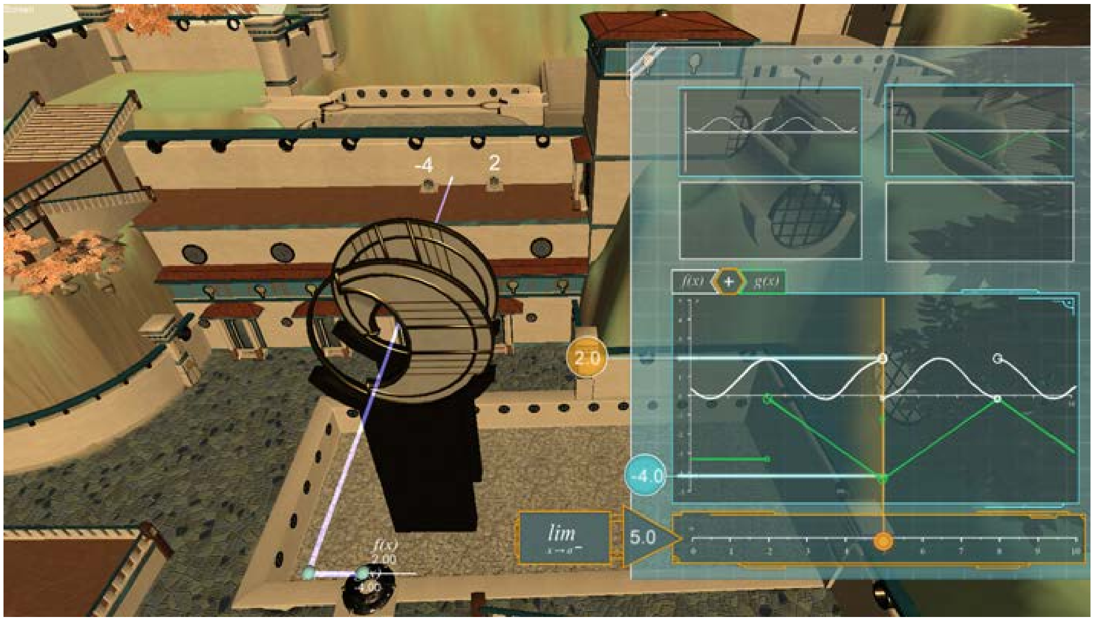

Figure 3- Variant: Limits, Challenge Interface

Source: "Triseum Gives AP and College Calculus Students Direct Access to Varian." *PRWeb,* 16 Mar. 2017, www.prweb.com/releases/2017/03/prweb14154225.htm.

The game environment is visually compelling, contains a narrative, and provides the player with visual and auditory clues about the goals and how to proceed. In addition, the interactive graph interface helps the player test their knowledge of Limits by manipulating the graph, trying different solutions, and get feedback by immediately seeing the results in the game world. Triseum claims that the game "…promotes conceptual understanding through direct interaction and immediate feedback in the game environment" ("Variant: Educational Game - Bringing Calculus to Life"). However, I believe this claim requires further investigation. As indicated earlier, conceptual understanding of a subject enables the learner to understand the importance of a mathematical concept and the contexts in which it might be useful. The game provides the player with multiple variations of problems related to Limits and offers an opportunity to practice. Yet, the problems and the mathematical concepts in focus do not have any meaningful connections to the game environment. The player's answer to a question does have an effect on the game environment, but the problem itself does not have any conceptual meaning within the game world. It is simply an obstacle, with some mathematical heft in it, that should be overcome to reach the next level. Replacing this obstacle with another type of challenge does not have any major effects on the game structure and narrative. The challenges incorporated in the game provide an opportunity for players to better practice but do not show the applications of the concepts in focus and how they could be used to solve problems in different situations. Whether or not *Variant: Limits* succeeds in improving students' conceptual understanding of the subject requires further research and investigation. It is important to mention that the game has been recently developed and, hence, there have not been many scholarly papers published about it to this date.

Chapter 3: Research Methodology

The main methodology used in the design and development process of this project is *design-oriented research* (Fallman, 2007). The prototyping process started in the very early stages of this research. Through an *iterative design* process, prototypes were created and presented to academic and industry experts for feedback. The feedback was used to refine the prototypes and further develop new ones which would go through further iterations. The latest version of these prototypes was put in front of grade 11 and 12 students for *playtesting*, once an approval from OCAD University's Research Ethics Board was received[4].

3.1. Design-Oriented Research

The term *design research* is increasingly used in industry and academia to describe different approaches, perspectives, and methods in the field of design (Faste & Faste, 2012). This term merges the two well-established practices of *research* and *design* and creates a meaningful concept. *Research* could be defined as "…a systematic investigation that establishes novel facts, solves new or existing problems, proves new ideas, or develops new theories" (Faste & Faste, 2012) and its intention is to "…produce knowledge and to seek the truth" (Fallman,

[4] More details in Appendix B

2007). *Design* on the other hand "…deals with the act of planning and communicating a course of action to others, usually through the creative exploration of an area of interest" (Faste & Faste, 2012). Hence, *design research* is equivalent to "…the investigation of knowledge through purposeful design" (Faste & Faste, 2012). Researchers have defined various categories of design research, as a very broad field, such as research *into/about* design, research *for* design, and research *through/by* design (Findeli, 1998; Frayling, 1993; Jonas, 2004; qtd. in Jonas, 2007). Fallman explores design research in the field of Human-Computer Interaction by distinguishing between *design-oriented research* and *research-oriented design*. According to Fallman, in *design-oriented research*, the main contribution is the knowledge created by studying the designed artifact and the process of creating it. The artifacts produced are in fact sketches or prototypes, rather than complete products, and act as means to get at knowledge, which is the final result (Fallman, 2007). In *research-oriented design*, on the other hand, the artifact, which in this case is closer to a finished and styled product, is the main outcome and contribution of the designer. This does not imply that knowledge is not generated through research-oriented design. Rather, the difference in purpose of the design activity creates different kind of knowledge; a knowledge that is not universal, but particular to its character (Fallman, 2007). Fallman uses the following diagram to describe the difference between the two conducts:

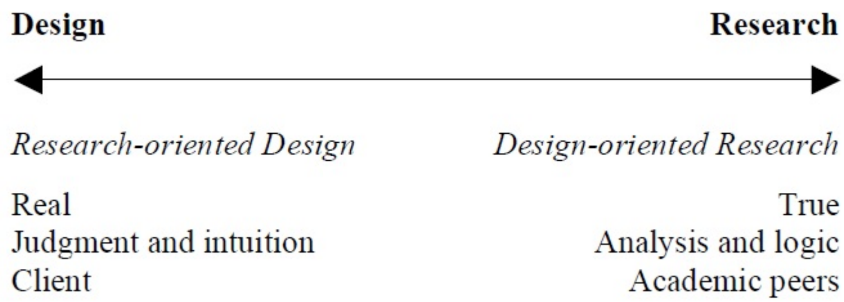

Figure 4- Research-Oriented Design vs. Design-Oriented Research

Source: Fallman, Daniel. "Why research-oriented design isn't design-oriented research: On the tensions between design and research in an implicit design discipline." *Knowledge, Technology & Policy* 20.3 (2007): 193-200.

The methodology used in this research has been *design-oriented research*. The goal has not been to develop a complete product based on the standards and preferences available in the game market, what Fallman refers to as the *Real*. Rather, the intention was to seek the *Truth* (Fallman, 2007) by developing prototypes and studying them in detail, to understand how video games could be used to help students better understand abstract symbolic mathematics. From the early stages, several prototypes have been developed based on research in the field of educational game design, specifically math educational games. The findings from the literature review and guidelines provided by experts such as Devlin, Gee, Coller, etc., as described in detail in chapter 2, were incorporated into the prototypes. The prototypes were tools to illustrate the research findings, in addition to artifacts that could be further studied and user-tested to discover new insights and verify my suppositions about learning through games.

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3.2. Iterative Design

Iterative design includes an iterative cycle of prototyping, user testing, analyzing, and refining based on the findings (Zimmerman, 2003). Prototypes should be tested by a group of users representing the potential audience of a product, early on during the development process and before settling on many ideas. This ensures that the product is aligned with the users' needs and preferences. Through this iterative process, designers could discover the answers to questions that arise during the testing and that they had not thought of before, which shows the power of iterative design as a research methodology (Zimmerman, 2003). When designing a game, chances are that the original design would change significantly during the design process. Because of the emergent nature of games, it is often very difficult to predict the behavior of a game before implementation, hence, the game design process is highly iterative (Dormans & Holopainen, 2017).

During this research process, a total of three distinct prototypes were developed through multiple iterations. The third prototype was developed over a longer period and went through more iterations. Each prototype was presented to advisors, industry experts, and peers for feedback, as soon as it reached a playable state, and was refined based on the feedback. A more detailed description of each prototype, how they were developed and how they changed over time, would be presented in chapter 4.

3.3. Playtesting

In the case of game design, iterative design is based on playtesting (Zimmerman, 2003), which is necessary to collect feedback from players and evaluate the effectiveness of the developed prototypes. In this project, the goal of playtesting was to examine the game's overall playability, user experience, visual appeal, and the quality of the interaction, in addition to outcomes related to learning, such as knowledge and skills acquisition. Assessing the learning outcomes requires running more and longer studies using control groups. Yet, considering the scope of this research, the results will be assessed through users' feedbacks only.

The main prototype incorporates concepts from grade 11 Functions course. Hence the target users were chosen from grade 11 and 12 students. Participants recruitment happened by sending invitation cards, in addition to posting online advertisements (figure 5). Participants were selected from various schools (public and private), with different teaching qualities, and located in neighborhoods characterized by both high and low socioeconomic status. Four of the participants were selected from *For Youth Initiative* center, located in western Toronto, and the other six were students of different schools located in downtown Toronto. In addition to secondary students, the game was also tested with peers and colleagues to provide feedback about the overall game experience and visual elements. Participants were selected based on their school grade only, regardless of their ethnicity or gender. Formal playtesting happened between January and March 2018 and took place at OCAD University, For Youth Initiative center, and Toronto Reference Library.

During the playtesting sessions, participants were asked to play the game, using a PC provided, and vocalize their thoughts as they play. Once they were done playing, a short interview session was conducted to gather the player's comments and suggestions. The questions asked during the interview were mainly concerned with the experience of the player, whether they found the game engaging, and how they compared the mathematical challenges embedded in the game with those of their textbooks. Below are some sample questions asked during the user testing session:

- Were the tasks and instructions given clear?
- Were you able to relate the task to a specific subject of your textbook?
- Did the game help you better understand the math concept it presents? If so, how?
- How was your experience of facing a math challenge different from classrooms?
- How does this game compare to other games that you enjoy?
- What did you find engaging about the game?
- What did you find disappointing about the game?
- Do you see yourself playing such a game frequently? If not, why?

A more comprehensive description of playtesting results will be presented in chapter 5.

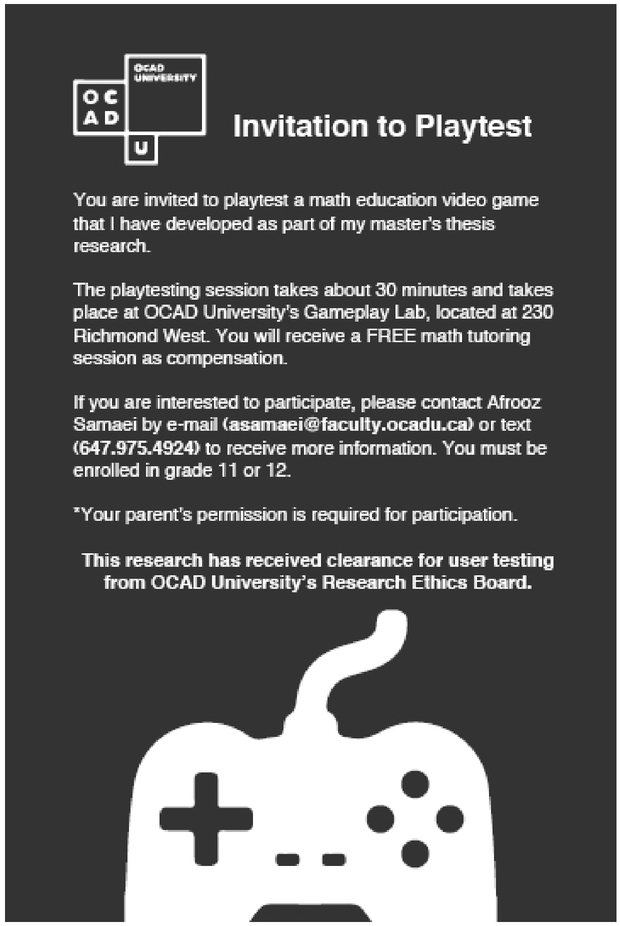

Figure 5- Invitation Card for Playtesting

Chapter 4: Prototypes

Over the course of this research project, three prototypes were developed. The first two prototypes were developed as initial experiments, over a shorter period of time, and went through fewer iterations. The third prototype is the major one supporting this written document and has gone through multiple iterations and been more formally user tested. This chapter illustrates the process in which the three prototypes were developed and formed over time. The prototypes have been developed using Unity game engine and C# programming language. The visual assets have been mostly taken from Unity Asset Store[5] and modified to suit the game. A few of the visual assets have been developed in 3ds Max modeling software.

4.1. Prototyping Process Overview

Building each prototype consisted of three separate but interwoven phases:

1. Designing the mathematical challenges
2. Designing the game environment and gameplay
3. Testing and refining the gameplay

[5] More details in Appendix C

As indicated earlier, the design of the game world would mainly depend on the mathematical challenges. Hence, I began by designing the mathematical problems. After coming up with the challenges, I started to experiment with different game environments and different ways I could embed the mathematical challenges in them. An important principle for me when designing the game has been incorporating the math challenges into the game environment such that they arise from the world, without seeming to be a separate add-on. So, the environment needed to match the mathematical challenges.

A common theme in all the three prototypes is navigating a three-dimensional environment using a car. My goal has always been to illustrate how certain mathematical concepts could be used in a real world. However, the concepts taught in high schools are relatively very basic and not comprehensive enough to be used in a real-world situation or accurate simulations of physical events. Hence, I created game worlds that are very simple representations of the physical world. The environments look casual and cartoony, yet they have enough connections and similarities to the real world to provide a familiar context. This could potentially help students understand how to use their knowledge within a visually appealing world that is not as complex as the real world but could provide situations for them to use their math knowledge.

Building the game world was a foundation for designing the gameplay and the game rules. Once everything was set up, I tested the gameplay and made modifications based on the user testing results. The design process took approximately three months in the case of the first prototype, two months for the second prototype, and six months for the last one. The first two prototypes were mainly user tested with peers and experts, whereas the third one was put in front of grade 11 and 12 students, as potential users of the final product. More details of the prototyping process will be discussed specifically for each prototype in the following sections.

4.2. Prototype 1

Figure 6 - First Prototype

The focus when developing the first prototype, as an initial exploration, was visualizing basic arithmetic operations within a game environment. The goal was to create a challenge that requires the player to perform simple arithmetic operations with positive and negative numbers, based on the settings of the game environment. The source of inspiration for this prototype was a mobile game called *Wuzzit Trouble,* developed by BrainQuake, which intends to increase student number sense (Kiili et al., 2015). In this game, positive and negative whole numbers and their combinations are visualized through a gear mechanism, which allows the players to get a sense of these numbers and simple arithmetic operations by turning the gears in different directions and in certain amounts (figure 7).

Figure 7 - Wuzzit Trouble

Source: "Wuzzit Trouble Extreme - Android Apps on Google Play." *Google*, Google, play.google.com/store/apps/details?id=com.brainquake.wuzzittroubleextreme.

The intention of developing the first prototype was to visualize the same algebraic models used in *Wuzzit Trouble*, using a different method of representation and within a three-dimensional world. In this prototype, the player drives a car from its origin to the destination, within a town. The town is built upon a grid of roads and the car can move along the grid only (figure 8). There are multiple ways to get to the destination, yet the player should find the route that leaves the car with the highest amount of gas when it arrives at the destination. As the car moves it loses a certain amount of gas. Alternatively, it can gain some of it back by picking up the barrels that are placed in certain positions within the town (figure 9). In the end, the player is rated with one, two, or three stars based on the amount of gas left in the tank (figure 10). Here is how the amount of gas changes:

- Moving along the vertical paths results in losing 40 units of gas (-40)
- Moving along the horizontal paths results in losing 20 units of gas (-20)
- Picking up a gas barrel results in gaining 60 units of gas (+60)

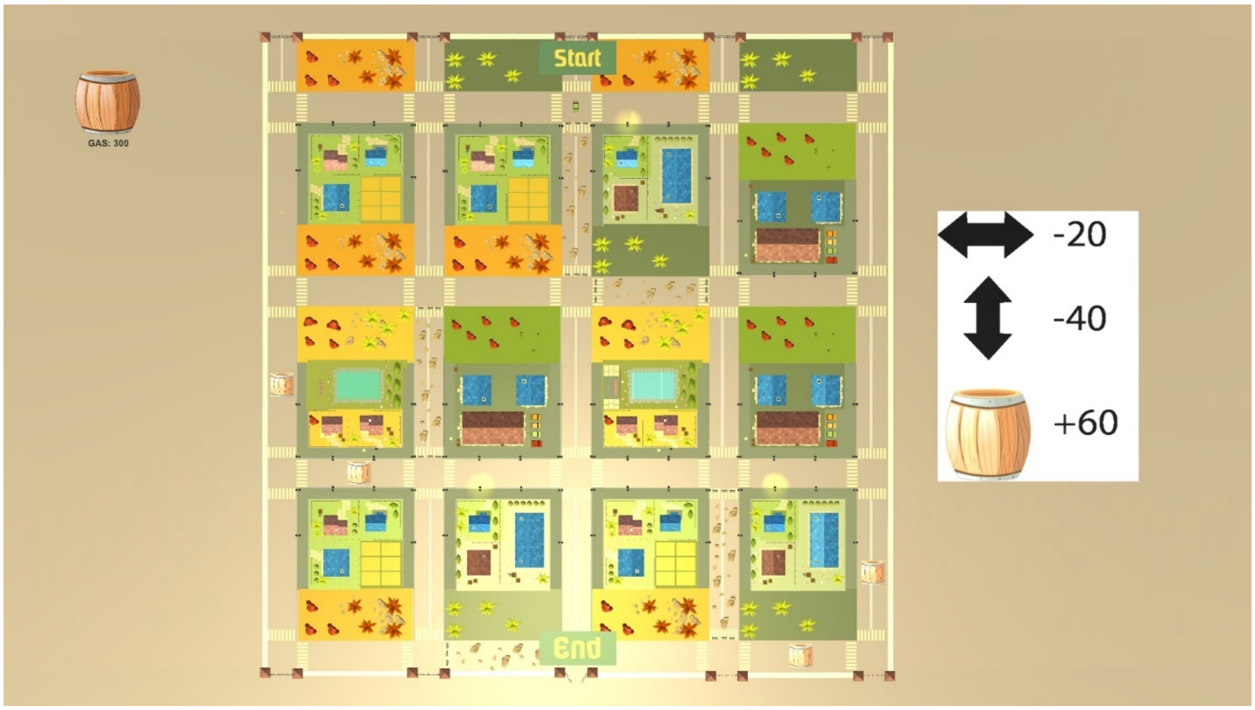

Figure 8 - First Prototype, Top View

Figure 9 - First Prototype, Picking Up Gas Barrels

Figure 10 - First Prototype, Scores

The players could play the game simply by trying different paths and seeing the results. However, by building a strategy from the beginning which requires doing some arithmetic operations they could save themselves from trying all possible routes.

By trying to compare potential routes through arithmetic operations, the player is, in fact, building a simple algebraic expression:

$$G = -20x - 40y + 60z$$

Where:

$x \rightarrow$ Total number of horizontal paths taken

$y \rightarrow$ Total number of vertical paths taken

$z \rightarrow$ Total number of gas barrels picked up

$G \rightarrow$ The amount of gas (By adding this value to the initial gas amount the remaining gas in the tank can be calculated)

The feedback that I received regarding this prototype was mostly concerned with the balance between educational goals and a fun gameplay. Although the visual assets were found compelling, the prototype needed improvements in terms of the level of enjoyment. In addition, it could benefit from a real-time feedback system that lets the players modify their decisions during gameplay, rather than building the whole strategy from the beginning. Developing the first prototype enabled me to decide on the area that I want to explore more, which is secondary level mathematics, and further narrow down the focus of my research.

4.3. Prototype 2

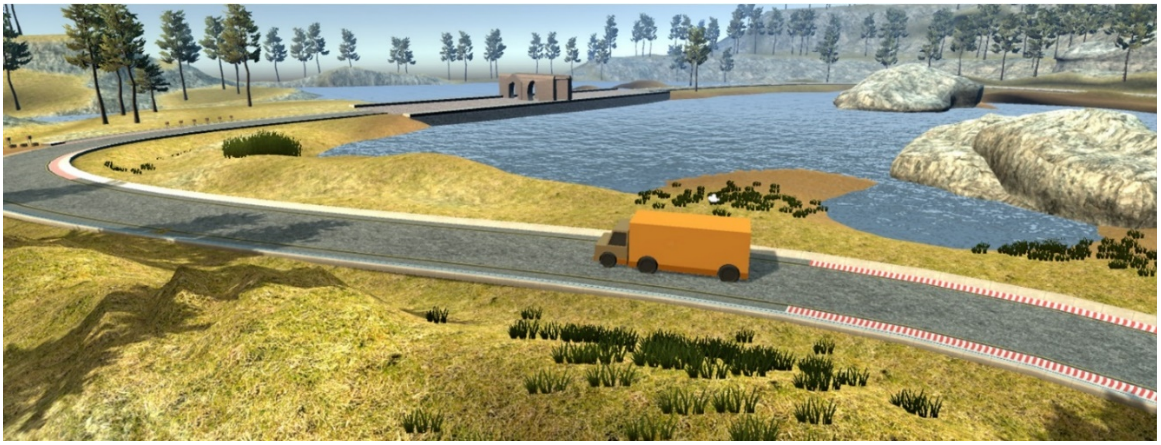

Figure 11 - Second Prototype

When designing the second prototype, my focus was to target high school students. The subject that I decided to focus on is Modeling with Quadratic Functions (polynomial functions of the second degree), which is an introduction to non-linear changes:

$$y = ax^2 + bx + c$$

Students are introduced to this subject in the second half of grade 10 and continue investigating it in more depth in grade 11. By grade 12, students have had enough exposure to quadratic functions and they start learning about polynomial functions of higher degrees:

$$y = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_2 x^2 + a_1 x + a_0$$

I decided to focus on this subject as it is covered considerably in the curriculum and is a basis of understanding curves and different behaviors of polynomial functions. The problem that I chose to embed in the game was taken from Grade 11 Functions by Nelson (p. 192):

> A tunnel with a parabolic arch is 12 m wide. If the height of the arch 4 m from the left edge is 6 m, can a truck that is 5 m tall and 3.5 m wide pass through the tunnel? Justify your decision.

I found this problem interesting as it requires students to think critically and analyze the situation from multiple perspectives. To solve the problem, students should decide where to place the truck compared to the arch, where to place the coordinate system compared to the arch and truck, how to find the equation of the arch, etc. This engages the students in a process of mathematical thinking, which involves deciding what details to ignore and what elements to keep in order to simplify and solve the problem. Hence, I found potentials in this question to be altered and investigated more by being embedded in a game world. This was the starting point for developing the second prototype. I began by designing tunnels with a parabolic arch and placed them within an environment that is visually appealing (figure 12). The environment consists of a track around a river, with tunnels placed on certain positions.

In this prototype, the player controls a truck. As the truck navigates the environment, it faces tunnels on the way. Some of the tunnels are not designed for the truck's dimensions. So, the player should choose the right path based on the given dimensions of the tunnels' entrance and the truck (figure 13). If the player chooses the wrong path, the truck gets damaged. In the end, the player is rated by one, two, or three stars based on the amount of damage to the truck.

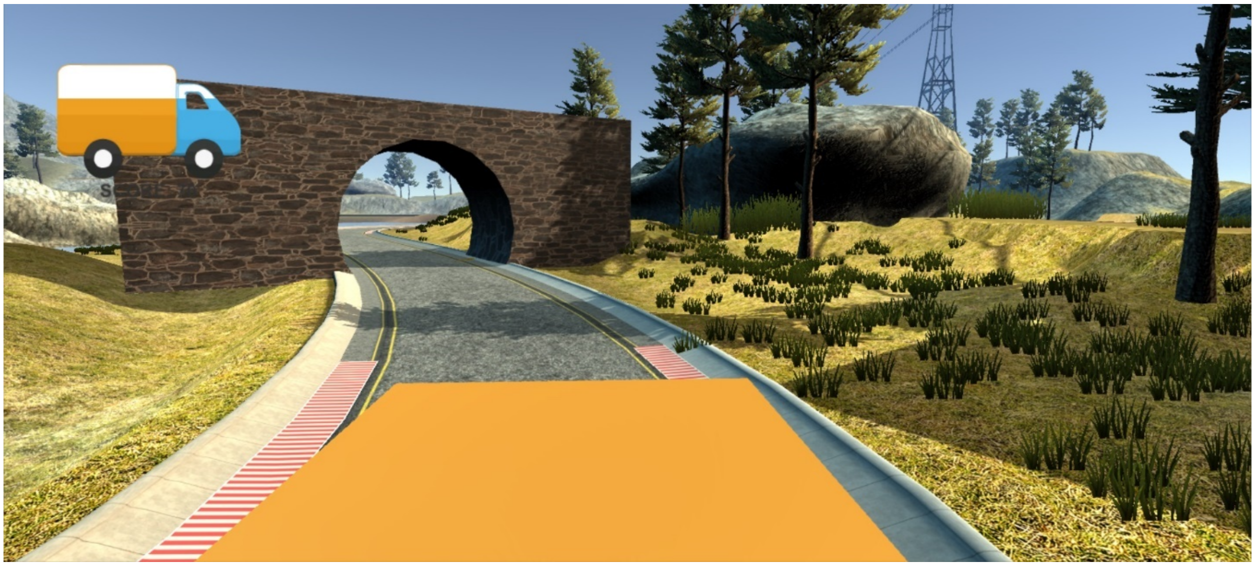

Figure 12 - Second Prototype, Passing through the Arch

Figure 13 - Second Prototype, Challenge

The main goal of developing this prototype, and the one after, is to help the players practice writing mathematical models and learn how to apply their knowledge of mathematics to solve problems similar to those in the real world. The key in writing models is one's ability to simplify a complex situation based on reasonable assumptions – in other words, to think mathematically. In this case, a truck, with all the details in its structure, is passing through a tunnel with a parabolic arch, within a three-dimensional environment. But, the situation can be simplified to a problem of finding the points of intersection between some lines, representing the truck's outer boundaries, and a parabola, representing the tunnels' entrance (figure 14).

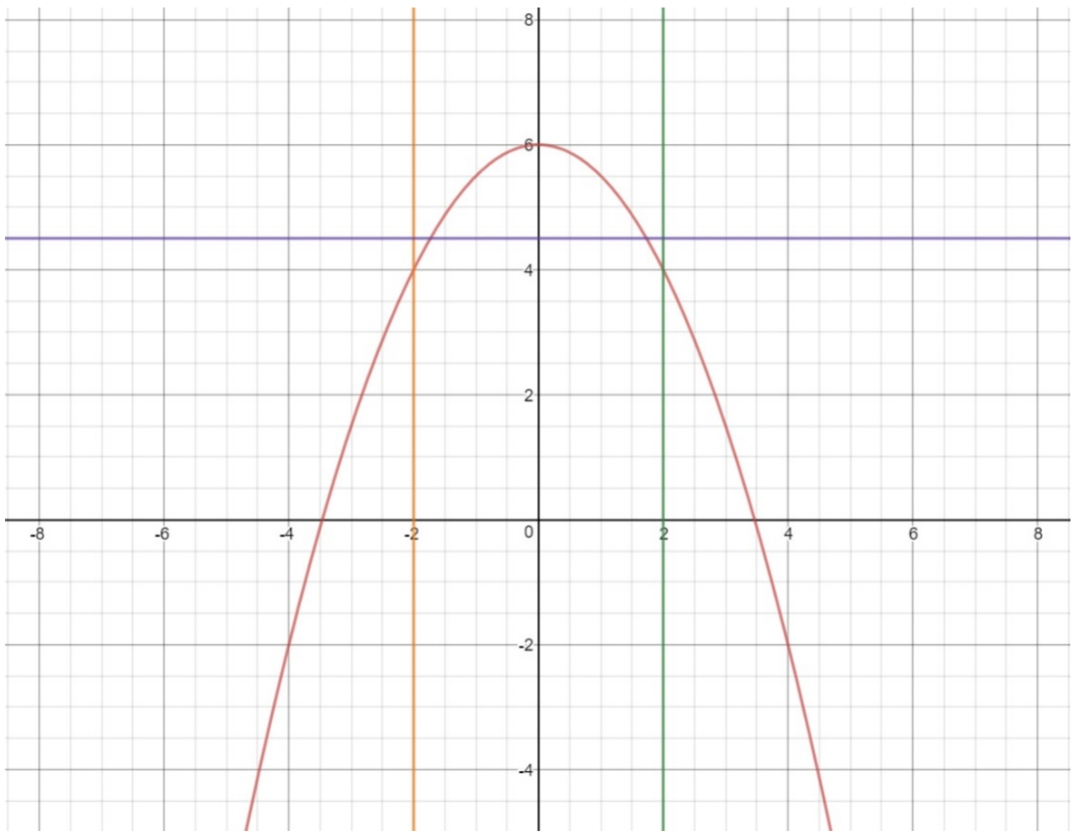

Figure 14 - A graphical representation of the game's challenge (Graphed with Desmos)

This is a problem that is very common in the textbooks and students can often solve without much difficulty, but only if it is phrased in this literal way:

Find the points of intersection between a line and a parabola with the following equations:

$$y = x + 1$$

$$y = x^2 - 4x + 3$$

Yet, transforming a real-life event with all its details and complexities to create such a simple problem is the main challenge for most students. This is what Devlin refers to as the “transfer problem”, namely one’s inability to take what they have learned in a class or a context and apply it in another class or in a real-world situation (Devlin, 2011, p. 74).

This prototype was presented at the CFC Media Lab to industry experts, and later to John Mighton, mathematician and the founder of JUMP Math. The comments regarding the concept were positive, overall. The idea of showing applications of mathematical concepts in a game world, the visual assets, and the embedded problem itself were found compelling. Yet, the prototype still needed improvements to enhance the gameplay experience and engagement. As mentioned by one peer, “…the biggest challenge for Afrooz is making her game truly engaging for children/teens. By creating a game, I believe Afrooz is not only competing with alternative ways of teaching math but also with the games popular in the age group she is designing for. Minecraft, Halo, Mario... these are what 'game' means to many teenagers”. Other comments suggested that the players need to be able to repeat a task, if they failed in their first try until they could solve the challenge. In addition, the game environment needed to be equipped with tools that allow the player to shift most of their calculations from paper to the game world and minimize the gap between the gameplay and mathematical calculations.

4.4. Prototype 3

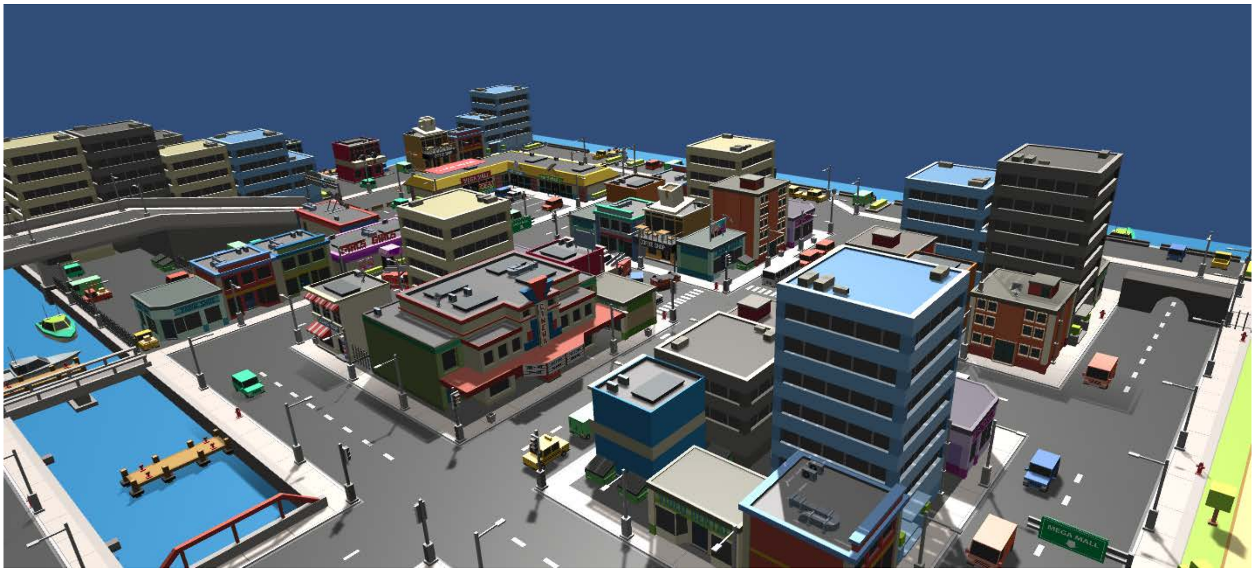

Figure 15 - Third Prototype

The third prototype, which is the main focus of this document, is built based on the findings from the first two prototypes. The main questions to be answered by building this prototype were:

- How can I integrate most of the mathematical procedures into the game?
- How can I provide instructions in case the player does not know how to solve a problem?
- How can I break the challenge into separate but related steps with an increasing level of difficulty?
- How can I increase players' interactions with the game world and ability to manipulate the game objects?
- How can I increase players' engagement?

The focus of this prototype is once again writing models with quadratic functions.

Figure 16 - Third Prototype, Navigating the Town

In this prototype, one of the main goals was to transfer most of the mathematical operations that the player needs to do to complete a challenge to the game environment. So, there was a need for an interface which allows some mathematical operations. In addition, this interface provides a space to incorporate instructions that would guide the players in a direction to solve a challenge, in case they are not able to do so. The interface consists of a graphing tool, using Desmos APIs (figure 17). This tool allows the players to graph the mathematical models they come up with in order to have a simple visual representation of the situation to potentially better understand it.

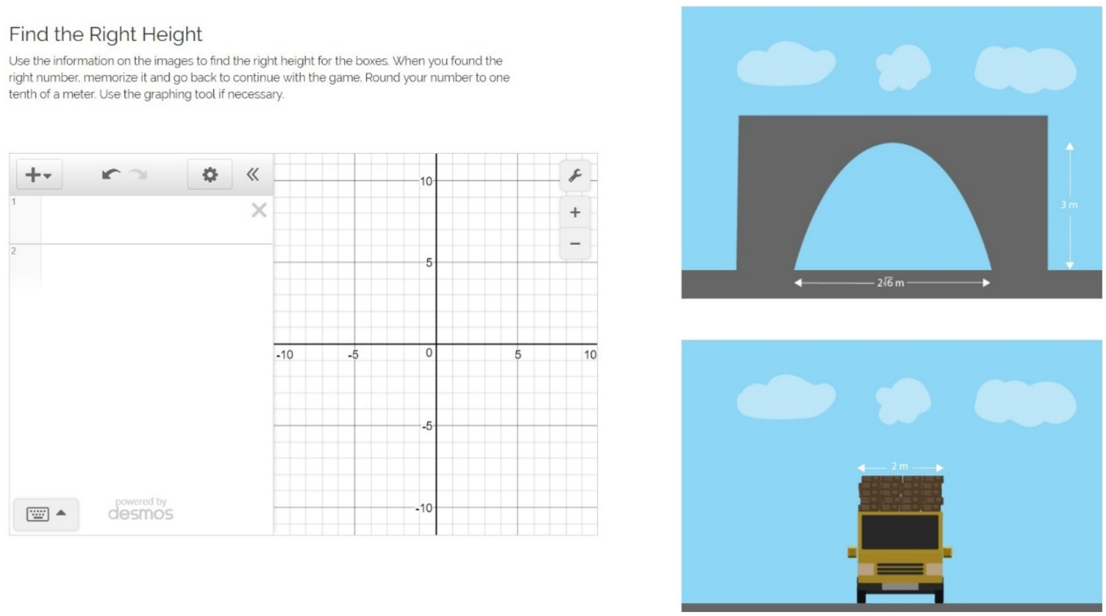

Figure 17 - Desmos Graphing Interface

The game environment consists of a town, where the player can navigate by driving a car. The task is to deliver some boxes to a destination. During this process, the players face a set of challenges, similar to those in the second prototype, that require them to use their knowledge of mathematics.

In the first version of this prototype, the car starts moving from an origin and is required to navigate the town, look for boxes, collect them, and deliver them back to the place where it began the game. On the way back, the car needs to pass through an arch-shaped tunnel (figure 18). Since the boxes collected and piled are higher than the safe height, the player is asked to calculate a right dimension for the piled boxes and unload some based on the calculated dimension. In the end, the players are rated with one, two, or three stars based on the number of boxes they manage to keep and deliver. The players could decide to unload all the boxes and safely pass through the tunnel. However, the highest point is achieved by carrying the maximum possible number of boxes. This scoring mechanism could potentially entice the player to analyze the situation and use their mathematical skills, rather than trying random numbers, to find the smallest margin that would allow them to safely enter the tunnel.

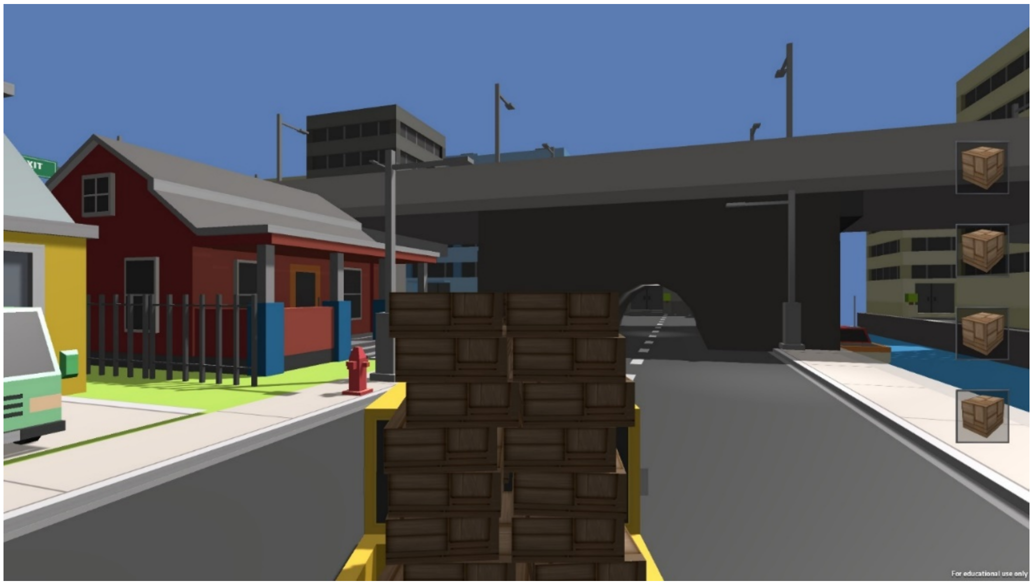

Figure 18 - Third Prototype, Passing through the Arch

By user testing this version of the prototype with colleagues and experts, it became apparent that the game environment was perhaps too open, to a degree that lets the player move freely and potentially get lost and never get a chance to face the main challenge. In other words, it seemed that the game's challenge was not evenly distributed in the environment. As it was suggested by Steve Engels, game design teacher of the University of Toronto, the challenges should allow the game environment to be gradually unlocked by solving a set of puzzles, preparing and leading the player to face the final challenge. This suggestion was incorporated in the second version of this prototype by breaking the previous challenge into four tasks with an increasing level of difficulty. Each task was designed in a way to potentially give the players some helpful clues to solve the next challenge and to gradually equip them with what they need to make progress.

First Challenge: The car needs to pass from under a road sign. In this case, the player should find the points of intersection between the boxes' boundaries and a horizontal line (figure 19).

Figure 19 - Third Prototype, First Challenge

Second Challenge: The car needs to pass through a bridge entrance that is formed by two slant beams. So, the player should find the points of intersection between the boxes' boundaries and two slant lines (figure 20).

Figure 20 - Third Prototype, Second Challenge

Third Challenge: The car needs to pass through a tunnel with a parabolic arch. So, the player should find the points of intersection between the boxes' boundaries and a parabola (figure 21).

Figure 21 - Third Prototype, Third Challenge

Final Challenge: The car needs to pass through a tunnel with a parabolic arch, one more time, but with different dimensions. The challenge would be finding the points of intersection between the boxes' boundaries and a parabola. Yet, this time no instructions are available, so the players should rely on their findings from the previous challenge.

The goal of breaking the challenges in this way was to engage the player in a process of finding the intersection of the car's boundary with different shapes, starting with a horizontal line, then two slant lines, and after that with two parabolas with different equations, which is the ultimate goal of this prototype (figure 22).

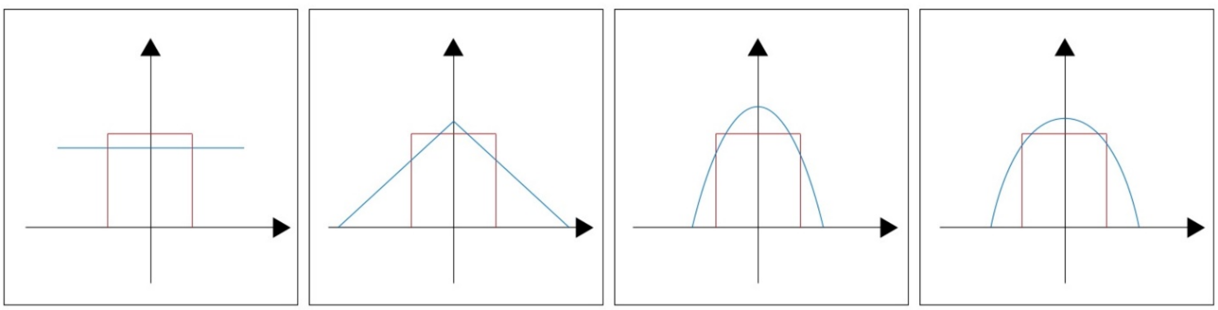

Figure 22 - Third Prototype, Simplified Representation of the Challenges

In the second version of the prototype, the game starts with the car being loaded with boxes. The task is to carry the boxes to a certain place in the city (figure 23). The car can still freely navigate the environment, but certain roads are blocked to help the player follow a path to face the four challenges step by step (figure 24). Once the car arrives at a position where each challenge is embedded, the interface described earlier pops up and provides the player with more details regarding the dimensions of the car and the entrances (figures 25). After performing mathematical calculations, players can find the maximum height to safely pass through the entrances. The more boxes they carry the higher their score would be. Players can always load or unload boxes, and the mathematical calculations allow them to find the optimum number of boxes to carry. In case they cannot solve a challenge on their own, they have the option to walk through the instructions provided.

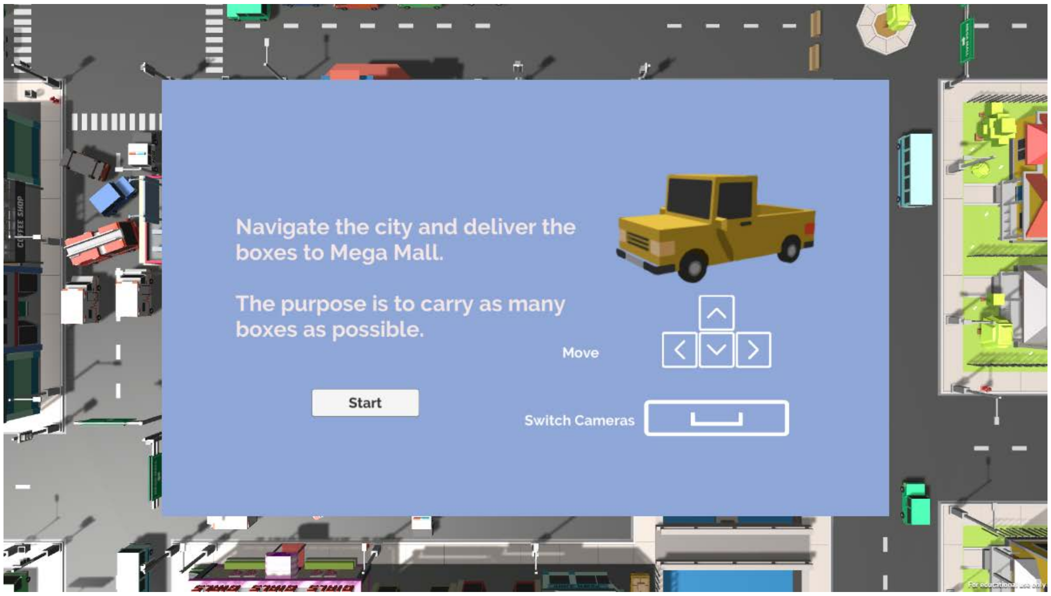

Figure 23 - Third Prototype, Task

Figure 24 - Third Prototype, Blocked Paths

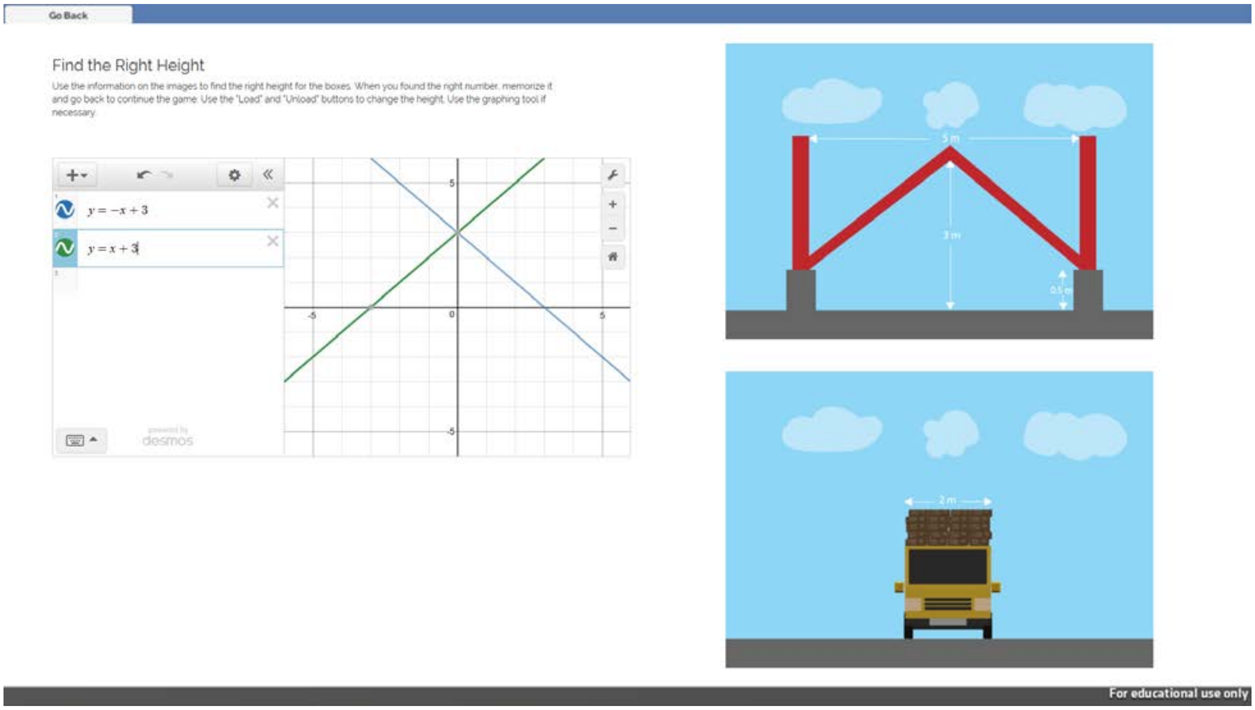

Figure 25 - Third Prototype, Getting the Dimensions

While it seemed necessary to use an interface that provides additional information and tools to help the players solve the problems, it raised some concerns about separating the player from the game environment for a long time. To improve this situation in the third version of the prototype the interface occupies half of the screen only, as opposed to a full-screen view, which could potentially help the player feel being less separated from the world as they watch the boxes being loaded or unloaded (figure 26).

Figure 26 - Third Prototype, Loading and Unloading Boxes

In the first three versions of the prototype, the players see their score at the end of the game and once they complete all the four challenges. They are rated with one, two, or three stars depending on how optimal their solutions are. The optimum height for the boxes, from the ground, in the four challenges are 2.1 m, 1.9 m, 1.9 m, and 2.4 m, respectively. Hence, if a player is able to find the best answer for each challenge their score would be 8.3, which is equal to the sum of the four numbers. This gives the player three stars at the end. Here are more details on the scoring mechanism:

Score = 8.3 → 3 Stars

7 < Score < 8.3 → 2 Stars

6 < Score ≤ 7 → 1 Star

Score ≤ 6 → No Stars

In the last version of this prototype, the players can see their score as they play and after completing each challenge. This change was made to make the scoring system more comprehendible. After completing each challenge players receive a star which is filled up based on their performance (figure 27). A full star is granted if the players manage to find the most optimal answer; that is the maximum number of boxes that allows them to safely pass through the tunnels.

Figure 27 – Scoring Mechanism

Chapter 5: Results and Findings

The prototypes developed throughout this research process have been play-tested by peers, instructors, academic and industry experts, in addition to students enrolled in grades 11 and 12. The third prototype went through a more rigorous process of testing-refining. Hence, this section is dedicated to the results of user testing the third prototype, as the results of playtesting the first two were briefly described in the previous chapter. To better analyze the results of playtesting, the feedback provided is divided into two main categories:

1. Gameplay Experience
2. Learning Outcomes

An analysis of the players' feedback from these two perspectives will be presented in the following sections.

5.1. Gameplay Experience

One feature of the prototype that was found appealing by all players was the game environment and the ability to navigate it by a car. The casual environment of the town was somehow in contrast to students' previous experience of the environments where they often encounter mathematical problems. “I wasn’t really expecting to see this!”, said one of the students when he first saw the game world. When I asked about what he was expecting to see, he explained that he was waiting for a screen with texts, describing some mathematical problems – a perception that many students have of a mathematical context.

Before starting the user testing, I asked each player if they identified themselves as a *gamer* or *non-gamer*. Having that factor in mind, I noticed a significant difference between the way the gamers and non-gamers played. The gamers spent more time navigating the environment, trying different paths, crashing the car into other cars, trying to push through the traffic jam, and mainly find a way to dodge the game challenges. Some of them were under the impression that they need to get to the destination as fast as possible, probably because this was in accordance with their perception of a game involving a car. They were also less patient with the math challenges and tried to find an answer by trying random numbers. The non-gamers, on the other hand, were more cautious as they moved the car, tried not to crash it, and followed the signs that lead them to the next challenge. When faced with a challenge, they took their time to read the question and used a pencil and paper to solve the problem. Most of them refused to follow the instructions and did their best to solve the challenges on their own.

When students were asked about their experience of solving the mathematical problems they all confirmed that they would prefer such a medium to their traditional textbooks. They indicated that this medium could help them see some of the applications of what they learn. In addition, some students seemed to be very interested in discussing the game and how it could be improved. One of the participants spent about half-an-hour talking, enthusiastically, about the new features that could be integrated into the prototype.

During the design process, I always faced an interesting tension between the entertainment and the educational aspects of the game. My supervisor's comments, as an expert in the field of game design, mainly encouraged less separation from the game environment and more direct interaction with the game objects. She believed that the pop-up interface containing information about the challenge or having to perform calculations on the side might negatively affect the gameplay experience. Yet I, as a tutor, found it necessary to include an interface in the game that bridges the game world to the textbook materials. John Mighton's comments, which will be discussed in more depth in the next section, suggested spending even more time within the interface as he found it necessary to give students some instructions before letting them solve the game's mathematical problems. Finding a common ground between these two opposing perspectives and keeping a balance between the education and entertainment were indeed the hardest challenge during the design process. Through this project, it became apparent to me that it is very difficult to embed abstract symbolic mathematics into a game without decreasing considerably the quality of the gameplay. Hence, more research needs to be done to find ways to solve this issue.

5.2. Learning Outcomes

The results of the playtesting showed that almost all students struggled to solve the challenges, despite that the math problems were derived directly from their textbook. Even those who were generally doing well in math with marks in the range of 80's and 90's spent a considerable amount of time thinking about the problem. They tried to experiment with different ways, which was desired, but often failed to fully complete the challenges. This was not surprising to me as I know, based on my experience, that even those students with high average marks in their math courses often have a very fragile understanding of the subjects. They are able to perfectly solve the problems that they face in their textbooks, which is a format that they are familiar with. However, if the environment and the format change or if the question is slightly twisted they fail to apply their knowledge to the new situation. In addition, a format that students are familiar with, in their textbooks, is learning a concept and solving related problems which immediately follow. Most of the time students know what method to use to solve a problem, only because they know the question is related to the subject that they just learned. For instance, when facing the third challenge in the game, one of the participants tried to use derivatives, although it was totally irrelevant to the situation, only because this was the subject that was being covered in his class at the time. All these confirm a lack of conceptual understanding of subjects that students learn in the classrooms.

When the third prototype was nearly done, I had a second meeting with John Mighton to discuss the prototype in terms of its educational aspects. Mighton's comments suggest that the problems embedded in the game need to be broken down further. As an educator, he provided feedback about how the instructions could be designed to help students better understand the quadratic functions and their properties. The point he insists on is that in any teaching tool we should break things down into the smallest possible pieces and think carefully about how many factors we are varying. "When there are lots of variables, [students] don't see what you want them to …Research shows you can develop ability in anything if you do deliberate practice. Deliberate practice has to be efficient. That means varying one thing at a time" (Mighton, 2018). For instance, in the case of learning the equation representing a quadratic function, students can experiment with changing the coordinates of the vertex (the turning point of a parabola) in the equation and understand how it affects the graph of the parabola. Once the vertex is completely understood, they could experiment with different values of the stretch factor to see how that factor changes the graph. As Mighton suggests, this process of modifying one thing at a time allows the students to recognize patterns and build mental representations that they can transfer to new situations. When I asked about the point at which we should leave students to be on their own and allow them to rely on their own creativity, rather than detailed instructions, he replied "Kids don't learn creativity by struggling. You have to keep them in a zone where they struggle productively. …The brain is not structured to think. You need to train it. You need to be guided to discover things and eventually you get to a point where you do things without guidance" (Mighton, 2018). As Mighton indicates, in the current educational settings the learning process is not well-scaffolded– that is teaching through small variations. Students struggle when faced with such problems, not because they have never seen them before, but because they have always been simply *told* how to solve the problem. They have never received a structured learning experience and hence when faced with a problem when so many things vary, they give up very quickly. "That is not to say that you do not want the kids to struggle, you do not want to push them, or you do not want to remove the steps, because sometimes they can handle it and there will be a point where you want all your lessons to be like that [more open, with fewer instructions], but if you want to bring every student along this is the efficient way to do it" (Mighton, 2018).

My goal when designing the game challenges was to entice the players to think analytically and creatively, rather than following instructions. I chose to work on a subject that students are introduced to in grade 10 and tested it on students in grade 12 since I assumed that they would be comfortable with the fundamental concepts of quadratic functions by then. Hence, the instructions provided in the game do not intend to teach students the basic concepts, but rather help them understand the methods they should use to solve the problems and remember the features of quadratic functions, in case they have forgotten. However, as Mighton notes, students might get confused and eventually give up if detailed instructions are not provided to them since they did not necessarily have a well-scaffolded learning experience in earlier grades.

Based on these user testing results and my discussions with John Mighton, the role of a teacher remains very important in this process. When students felt lost while solving a problem during the playtesting, they were able to continue after I gave them small hints. Hence, the game is not a replacement for teachers. It is rather a tool that empowers and complements them by providing situations where students can better understand the applications of a mathematical concept. The teacher scaffolds the learning and the game, as a homework tool, supports the learning experience. My role as a math tutor has always been to assist students with what they have previously learned in classrooms and provide context for them to understand how to use it. I suggest that the prototypes developed in this project, as a series of tools that could assist the students with understanding a concept more profoundly, strengthens my role as a math tutor.

The playtesting results proved to me, once again, how much effort should be made to enhance mathematical education. Unfortunately, what students learn mainly at schools is to follow instructions that are meaningless to them, without receiving enough contexts of how they could use what they learn. This emphasizes the importance of learning how to think mathematically, as indicated by Devlin, and how to build mathematical representations of a problem, rather than blindly following mathematical procedures. Students could easily forget the steps required to solve a problem, yet it is very unlikely to forget how to think like a mathematician if they are trained to do so.

5.3. Future Iterations of the Prototypes

In the future iterations, instructions should be broken down further and made available for those who require more information. As suggested by Mighton, including an enriched version, which contains more complex problems compared to the regular version, could ensure that the game operates within the learner's "regime of competence" (Gee, 2007, p. 68) as it prevents the stronger students to become bored and the weaker ones to become overwhelmed. In addition, the game could benefit from multiple levels, each showing one application of quadratic functions in different contexts. The second level of the third prototype, which is currently under development, focuses on quadratic functions as a tool that could represent the trajectory of an object that is thrown in the air. Finding the information regarding the movement of a projectile is a very common question in the textbooks when the quadratic functions are taught. In the second level of the game, the object moving on a parabolic path would be the car itself, as the player tries to jump from one point of the environment to another when there are obstacles on the way (figure 28). Students need to find the right equation of a parabola which ensures the car lands safely on the other side, given that it moves along that path.

Figure 28 - The car jumps and moves along a parabolic path to get from one part of the environment to the other

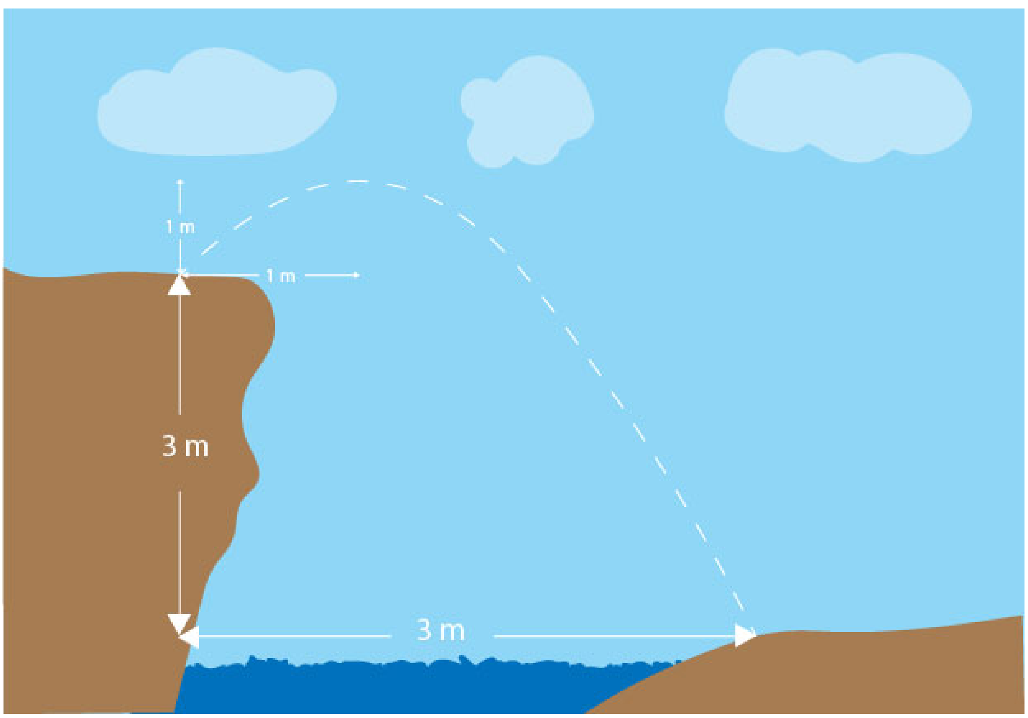

Figure 29 - The parabolic path that ensures the car lands safely on the other side

Moreover, the game could benefit immensely from a game analytics tool which gathers data regarding players' behaviors during the gameplay. The data could be further analyzed to enhance the game experience and modify the consecutive challenges based on the player's performance on previous ones. Tailoring the game to each player ensures that they reach a state of *flow* (Csikszentmihalyi, 1990), which is achieved by "increasing the level of challenge as the individual's skill level increases so there is a dynamic tension between the states of boredom and frustration" (Annetta, 2010).

As indicated earlier, performing a rigorous assessment of the impacts of playing such games on students' learning requires running studies over a longer period. These studies should include a control group to make comparisons between students who were exposed to the game and those who followed a traditional method. In addition, the criteria for assessment should not be limited to students' performance on timed tests and more assessment tools need to be designed and developed. Future works would focus more on the assessment mechanisms.

Chapter 6: Conclusion

This chapter provides a brief review of the previous chapters to conclude the thesis. It also further discusses the next steps and future research that could expand this research project.

6.1. Review

The first chapter of this thesis illustrated the main problem that this research intends to address and why discourse on this subject matter is important. Mathematics, the science of patterns (Devlin, 2000, p. 35), is a crucial skill to acquire for today's students as the future scientists, technologists, and innovators who should lead forward a country's development and solve complicated problems. Yet, many developed countries with high rates of industrial and technological advancements are concerned about students' lack of proficiency and interest in STEM fields since it could threaten their position and decline their competitiveness in the global economy. The number of science and engineering jobs has been growing significantly since 1980 in the United States for instance, but the number of people trained to fulfill these positions is decreasing (Shaffer, 2006, p. 1).

Learning mathematics is indeed difficult and requires lots of effort and persistence from the learners' side, and lots of research on developing innovative ways of teaching from the educators' side. Since the twentieth century, many researchers and scholars have been engaged with theorizing how humans learn and what are best pedagogic ways to improve learning, but it is important to remember that education is an empirical science (Devlin, 2011, p. 196). Not only should different pedagogic methods be constantly tested, the studies themselves should be carefully designed to identify what is being measured, what are the constant parameters from one study to the other, and what is being altered. In addition, the change in teachers and students' behaviors when participating in a study, compared to normal circumstances, must be considered (Devlin, 2011, p. 196). These complexities increase the need for ongoing research along with people with sufficient knowledge, skills, and enthusiasm willing to embark on this journey and contribute to this field.

This research intended to be a contribution to innovation in mathematics education, by focusing on video games and using the technology's unique features to motivate students to practice their previously acquired skills through a different medium. Video games are great tools for learning. Even those games which are not specifically designed for educational purposes offer an environment to players to face challenges, build strategies, form hypotheses, experiment with different solutions, safely and iteratively, and by doing all these learn new skills. Video games allow experiential, active, and critical learning and enable the players to proceed, and learn, based on their own pace and constantly receive feedback from the consequences of their actions and revise their future decisions.

The specific goal of this research was to answer the question of "How could digital games be used to help secondary school students practice applying abstract symbolic mathematical concepts described in their textbooks to solve problems?", through developing a series of prototypes, playtesting them, and analyzing the results to develop future prototypes. The main intention when developing prototypes has been to help high school students use their algebraic knowledge to practice writing mathematical models for real-life events and solve related problems. The subject chosen for the prototypes was quadratic functions, as students' first exposure to non-linear changes. The research methodologies used to develop prototypes were design-oriented research, iterative design, and playtesting. The prototypes were developed based on initial research on good principles of learning and affordances of video games in this regard. One of the main literatures used in this regard is the book *What video games have to teach us about learning and literacy*, by James Paul Gee. The main reason behind citing this work quite often in this paper is because it is a distillation of research on education, learning theories, psychological aspects of video games, visual literacy, etc. Although some criticize Gee for relying heavily on theories from his own field (literacy studies) rather than having a closer look at the literature on video games and learning (Egenfeldt-Nielsen, 2006), his texts proved to be a valuable resource to use in this research, considering their conciseness, clarity, and direct relevance to this thesis topic.

The developed prototypes went through multiple iterations and were user tested with different groups. The playtesting sessions along with short interviews were run to evaluate students' overall experience and their feedback on the mathematical challenges. Playtesting results showed that all the participants found the game environment visually appealing and enjoyed navigating it. They all confirmed that they would like to have access to such a medium in their classrooms to practice mathematics. However, more work needs to be done to improve the gameplay experience and entertainment aspects of the game, while maintaining the educational aspects. Most students found the game challenges difficult, compared to the questions in their textbooks and struggled to connect the problem to a specific concept in their textbook. This showed a lack of conceptual understanding of the subjects and the ability to apply them in a new context that they are not familiar with and emphasizes the importance of building new tools and finding innovative ways to fix this problem.

Although students struggled with solving the problems, they could continue with the task once they received brief instructions. This emphasizes the important role of a teacher as a facilitator in this process. The intention behind proposing such medium in this thesis has not been to replace teachers or classroom time with a digital game. Teachers need to scaffold the learning for students and provide additional help if they struggle to solve a problem. The game, on the other hand, provides an environment where students can practice what they have learned, by embedding abstract concepts within a concrete situation and provide a context for students to put into practice what they have previously learned. The game could provide the incentive for students to practice mathematics, in an environment where they can test their solution to a problem and where making mistakes does not have the costs and consequences of failing an exam. The teacher and the game have complementary roles and should work in symbiosis with each other. The focus when developing the prototypes has been centered on maintaining a balance between the two perspectives of 'progressives' who believe that situated embodied experience is crucial (Gee, 2005) and the 'traditionalists' who think that students need guidance and scaffolded learning and cannot be left on their own (Kelly, 2003, qtd. in Gee, 2005).

Lastly, it is important to note that there are many barriers preventing the implementation of video games into educational settings. Apart from the serious challenge of embedding subjects such as abstract symbolic mathematics into video games, one of the main obstacles of empowering classrooms with such tools is the cost. As Gee suggests, the cost here is not just monetary. “It is the cost, as well, of changing people’s minds about learning – how and where it is done. It is the cost of changing one of our most change-resistant institutions: schools” (Gee, 2005).

6.2. Future Research

In his book, *How Not to Be Wrong: The Power of Mathematical Thinking,* Jordan Ellenberg discusses some mathematical concepts from school mathematics outside of their familiar territory and illustrates how those concepts are tied to our daily life activities. For instance, he briefly talks about parabolas and compares them with lines to distinguish between thinking *linearly* or *nonlinearly*. Thinking linearly means that changes in one variable always increases or decreases the value of the other variable (changing according to a linear function). For instance, thinking that increasing the price of an item would always increase the profit, or vice versa. Thinking nonlinearly, on the other hand, indicates that changes in one variable might increase or decrease the value of the other variable (changing according to a quadratic function or a polynomial of a higher degree). For instance, increasing the price to a certain amount would increase the profit, but raising it more would result in losing money. This means “which way you should go depends on where you already are” (Ellenberg, 2014, p. 24) as there is always an optimum point (the maximum or minimum point on a parabola which is also referred to as the vertex). Applying this form of thinking in daily life helps us understand that having more (or less) of something does not necessarily lead to better results. Most changes are nonlinear, and hence there is always an optimum point.

Ellenberg divides the mathematical universe into four quadrants and shows were the concepts in his book, such as the example above, stand:

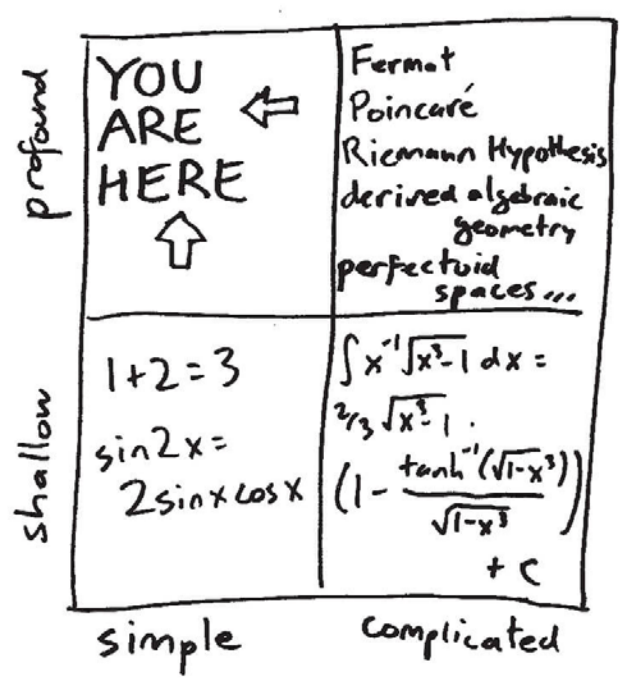

Figure 30 - Ellenberg's Quadrants of Mathematics

Source: Ellenberg, Jordan. *How not to be wrong: the power of mathematical thinking*. The Penguin Press, 2014. p.15

In the current education systems, mathematical concepts mostly cover the bottom-left and the bottom-right quadrants. The two quadrants differ in difficulty level and the time required to master them, but neither of them contains much conceptual heft (Ellenberg, 2014, p.15). The upper-right quadrant concerns professional mathematicians and those who pursue mathematics in their post-secondary education. Ellenberg focuses on the upper-left quadrant, as an area that offers mathematical ideas that one can engage with "…directly and profitably …They are the go-to tools on the utility belt, and used properly they will help you not be wrong" (Ellenberg, 2014, p. 16).

In my perspective, what students expect to receive from their mathematics education are concepts from this quadrant, which they could use directly in their daily activities. Not seeing more of these applications often result in frustration and confusion. Unquestionably, one needs to master the fundamental skills covered in the lower quadrants to later push forward their knowledge toward more profound concepts. However, by trying to show students how the skills they have accumulated to date could be applied to enhance an aspect of their life, rather small, they might appreciate mathematics and potentially become more enthusiastic about learning it. The mathematical problems embedded in the prototypes during this research were very simple engineering problems, which might not interest all students. Focusing on concepts that more students can feel connected to and embedding them in the future game prototypes could potentially increase the hopes of making mathematics seem more friendly and useful to students.

This is by no means an easy task. The enormous complexity of this challenge was proven to me personally during this research journey. Video games have indeed lots of potentials to enhance mathematics education. Research in this field requires collaboration among educators, game designers, and developers to ensure all aspects of the game are given enough attention. This thesis was for me an initial step forward to further investigate this field and explore ways to incorporate mathematical concepts, specifically more “simple/profound” ones, into video games.

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APPENDICES

Appendix A – Thesis Exhibition (April 2018)

Figure 31 - Thesis exhibition at OCAD University, April 2018

Link to screen captures of the prototypes: https://www.afroozsamaei.com/work#1

Appendix B – Research Ethics Board Documents

APPROVAL LETTER

December 01, 2017

Dr. Emma Westecott
Faculty of Liberal Arts & Sciences & School of Interdisciplinary Studies
OCAD University

File No: 101131
Approval Date: December 01, 2017
Expiry Date: November 30, 2018

Dear Dr. Emma Westecott, Ms. Afrooz Samaei

The Research Ethics Board has reviewed your application titled 'Video Games as a Tool for Learning Abstract Symbolic Mathematics'. Your application has been approved. You may begin the proposed research. This REB approval, dated December 01, 2017, is valid for one year less a day: November 30, 2018. Your REB number is: 2017-57.

Throughout the duration of this REB approval, all requests for modifications, renewals and serious adverse event reports are submitted via the Research Portal.

Any changes to the research that deviate from the approved application must be reported to the REB using the amendment form available on the Research Portal. REB approval must be issued before the changes can be implemented.

To continue your proposed research beyond November 30, 2018, you must submit a Renewal Form before November 23, 2018. REB approval must be issued before research is continued.

If your research ends on or before November 30, 2018, please submit a Final Report Form to close out REB approval monitoring efforts.

If you have any questions about the REB review & approval process, please contact the Christine Crisol Pineda, Manager, REB secretariat.

If you encounter any issues when working in the Research Portal, please contact our system administrator.

Sincerely,

Nancy Snow
Acting Chair, Research Ethics Board

INVITATION FORM

Date:

Project Title: Learning Abstract Symbolic Mathematics Through Digital Games

Principal Investigator:

Afrooz Samaei

OCAD University

E-mail

Phone Number

Faculty Supervisor:

Emma Westecott, Associate Professor

Faculty of Liberal Arts and Sciences

OCAD University

E-mail

Phone Number

INVITATION

You are invited to participate in a study that involves research. The purpose of this study is to understand how digital games could be used to facilitate learning mathematics for high school students. A prototype game will be developed as part of this research. In order to make sure that the final game meets the needs of the players and provides an enjoyable experience to them, we need users to participate in the research, playtest the game, and provide feedback.

WHAT'S INVOLVED

As a participant, you will be asked to playtest the prototype, which is a computer game, and provide feedback about your experience of playing the game. The user testing process takes place in the gameplay lab at OCAD University located at 230 Richmond West, and involves playing the game on a computer followed by a short interview to hear your opinions and feedback. The whole session will take approximately 30 minutes of your time. As a token of appreciation for your time, you will be offered a one-hour math tutoring session, free of charge.

With your agreement, we would like to contact you again in about 4 weeks to invite you for a second user testing session and to answer another set of similar questions. You may decide at that time whether or not you wish to participate in that part of the study.

POTENTIAL BENEFITS AND RISKS

By participating in the user testing session, you will get a chance to understand some of the applications of the mathematical concept that you have previously learned from your textbooks within a game environment and potentially better understand a concept that you might struggle with. There are no known or anticipated risks associated with participation in this study. The user testing process will be an individual session with the researcher being present in the room. The intention is to provide a comfortable environment for you to solve the challenges without the fear of being judged by others based on your performance.

CONFIDENTIALITY

You will be interviewed after playtesting the game in order to provide feedback. All information you provide is considered confidential and in possession of the researcher and her advisor; your name will not be included or, in any other way, associated with the data collected in the study. Furthermore, because our interest is in the average responses of the entire group of participants, you will not be identified individually in any way in written reports of this research.

Data (audio / video) collected during this study will be stored on a password protected computer in a reasonably secure location. Data (audio / video) will be kept for a year after which time project archiving will be deleted. Access to this data will be restricted to Afrooz Samaei and Emma Westecott.

VOLUNTARY PARTICIPATION

Participation in this study is voluntary. If you wish, you may decline to answer any questions or participate in any component of the study. Further, you may decide to withdraw from this study at any time, or to request withdrawal of your data (prior to data analysis which starts a week after the user testing session), and you may do so without any penalty or loss of your compensation.

PUBLICATION OF RESULTS

Results of this study will be published in the form of a student thesis and will be presented in the thesis defense session. In any publication, data will be presented in aggregate forms. This means we gather feedback from a group of participants, regardless of who provided it, and use the overall results to improve the game experience. Quotations from interviews will not be attributed to you without your permission.

The final thesis document will be available to view on OCAD University Open Research Repository webpage, under the category of Digital Futures 2018, after completion of the project in late April. You will also be given a copy of the final game.

CONTACT INFORMATION AND ETHICS CLEARANCE

If you have any questions about this study or require further information, please contact the researcher, Afrooz Samaei, or the faculty supervisor, Emma Westecott, using the contact information provided above. This study has been reviewed and received ethics clearance through the Research Ethics Board at OCAD University (REB number: 2017-57.). If you have any comments or concerns, please contact the Research Ethics Office.

VIDEO/AUDIO RECORDING

In order for us to better analyze the results we would like to ask your permission to video/audio record the playtesting session. We will video record the screen only. You have the right to reject video recording without any penalty or loss of your compensation.

In addition, as indicated earlier you would be asked to participate in a short interview, after playing the game, to share your experience and thoughts. With your permission, we would also like to audio record the interview session. The recorded audio would be used by the researcher and her supervisor to further analyze the results. You have the right to refuse to be audio recorded without any penalty or loss of your compensation.

CONSENT FORM

I agree to participate in this study described above. I have made this decision based on the information I have read in the Invitation Letter. I have had the opportunity to receive any additional details I wanted about the study and understand that I may ask questions in the future. I understand that I may withdraw this consent at any time.

Video recording the playtesting session

☐ I would permit video recording the screen only, with audio.

☐ I would permit video recording the screen only, without audio.

☐ I do not permit video recording.

Audio recording the interview session

☐ I would permit audio recording my interview session.

☐ I do not permit audio recording.

Name: ________________________

Signature: ________________________ Date: ________________________

Thank you for your assistance in this project. Please keep a copy of this form for your records.

PARENTAL PERMISSION FORM

Date:

Project Title: Learning Abstract Symbolic Mathematics Through Digital Games

Principal Investigator:

Afrooz Samaei

OCAD University

E-mail

Phone Number

Faculty Supervisor:

Emma Westecott, Associate Professor

Faculty of Liberal Arts and Sciences

OCAD University

E-mail

Phone Number

DESCRIPTION

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With your agreement, we would like to contact your child again in about 4 weeks to invite her/him for a second user testing session and to answer another set of similar questions. You may decide at that time whether or not you wish to give permission for participation in that part of the study.

POTENTIAL BENEFITS AND RISKS

By participating in the user testing session, participants will get a chance to understand some of the applications of the mathematical concept that they have previously learned from their textbooks within a game environment and potentially better understand a concept that they might struggle with. There are no known or anticipated risks associated with participation in this study. The user testing process will be an individual session with the researcher being present in the room. The intention is to provide a comfortable environment for your child to solve the challenges without the fear of being judged by others based on their performance.

CONFIDENTIALITY

Participants will be interviewed after playtesting the game in order to provide feedback. All information they provide is considered confidential and in possession of the researcher and her advisor; participants' name will not be included or, in any other way, associated with the data collected in the study. Furthermore, because our interest is in the average responses of the entire group of participants, they will not be identified individually in any way in written reports of this research.

Data (audio / video) collected during this study will be stored on a password protected computer in a reasonably secure location. Data (audio / video) will be kept for a year after which time project archiving will be deleted. Access to this data will be restricted to Afrooz Samaei and Emma Westecott.

VOLUNTARY PARTICIPATION

Participation in this study is voluntary. Participants may decline to answer any questions or participate in any component of the study. Further, they may decide to withdraw from this study at any time, or to request withdrawal of their data (prior to data analysis which starts a week after the user testing session), and they may do so without any penalty or loss of their compensation.

PUBLICATION OF RESULTS

Results of this study will be published in the form of a student thesis and will be presented in the thesis defense session. In any publication, data will be presented in aggregate forms. This means we gather feedback from a group of participants, regardless of who provided it, and use the overall results to improve the game experience. Quotations from interviews will not be attributed to participants without their permission.

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CONSENT

I agree to give permission to my child to participate in the study described above. I have made this decision based on the information I have read in this form. I have had the opportunity to receive any additional details I wanted about the study and understand that I may ask questions in the future.

Name: ______________________________

Signature: ______________________________ Date: ______________________________

Thank you for your assistance in this project. Please keep a copy of this form for your records.

PLAYTESTING QUESTIONNAIRE

- Do you play video games often? How many hours a day?
- What are your favorite games?
- Have you previously played any educational games? Do you have any favorites?
- How are you doing in math? How is your average?
- Do you like math? Why or why not?

- Were the instructions given at the beginning clear?
- Were the given tasks in the game clear?
- Did you understand the mechanism of getting high scores?
- Were you able to relate the task to a specific subject of your textbook?
- Did the game help you better understand the math concept it presents?
- If you were not willing to complete the game, please explain your reasons.
- How does this game compare with other games that you enjoy?
- What did you find engaging about the game?
- What did you find disappointing about the game?
- What elements could be added to the game to make it more engaging?
- Do you see yourself playing such a game frequently? If not, why?

- What do you think about such a platform to practice math in general?
- Would you prefer practicing math with a game or it wouldn't make much difference?

Appendix C – Unity Assets

Adventure Game Tutorial - Developed by Unity– Used in the third prototype

Source: "Adventure Game Tutorial." Unity, unity3d.com/learn/tutorials/projects/adventure-game-tutorial.

Lake Race Track - Developed by NIANDREI – Used in the second prototype

Source: "Lake Race Track." *Asset Store*, assetstore.unity.com/packages/3d/environments/roadways/lake-race-track-55908.

Simple Town - Cartoon Assets - Developed by SYNTY Studios – Used in the third prototype

Source: "Simple Town - Cartoon Assets." *Asset Store*, assetstore.unity.com/packages/3d/simple-town-cartoon-assets-43500.

Stylized Simple Cartoon City - Developed by AREA730 – Used in the first prototype

Source: "Stylized Simple Cartoon City." *Asset Store*, assetstore.unity.com/packages/3d/environments/urban/stylized-simple-cartoon-city-50095.

WWebView - Developed by ICODES STUDIO – Used in the third prototype to open webpages containing dimensions, instructions, and Desmos Graphing Tool

Source: "WWebView." *Asset Store*, assetstore.unity.com/packages/tools/network/wwebview-97395.